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ABSTRACT

In this module the student will learn about parallel RL (resistive-inductance), RC (resistive-capacitive), and RCL (resistive-capacitive-inductance) circuits and the conditions that exist at resonance. The module is divided into six lessons: solving for quantities in RL parallel circuits; variational analysis of RL parallel circuits; parallel RC and RCL circuits; parallel resonance; effective resistance in parallel RL circuits; and parallel resonance experiments. Each lesson consists of an overview, a list of study resources, lesson narratives, programed instructional materials, and lesson summaries. (Author/BP)

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BASIC ELECTRICITY AND ELECTRONICS INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

PARALLEL AC RESISTIVE-REACTIVE CIRCUITS

Study Booklet

BUREAU OF NAVAL PERSONNEL

January 1972

CE002 588

OVERVIEW

MODULE FOURTEEN

PARALLEL AC RESISTIVE-REACTIVE CIRCUITS

In this module you will learn about parallel RL, RC and RCL circuits and the conditions that exist at resonance.

For you to more easily learn the above, this module has been divided into the following six lessons:

Lesson I. Solving for Quantities in RL Parallel Circuits . . . .
Lesson II. Variational Analysis of RL Parallel Circuits . . . . .
Lesson III. Parallel RC and RCL AC Circuits . . . .
Lesson IV. Parallel Resonance . . . .
Lesson V. Effective Resistance in Parallel RL Circuits . . . . .
Lesson VI. Parallel Resonance Experiment . . . .

TURN TO THE FOLLOWING PAGE AND BEGIN LESSON I.

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BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON I

Solving for Quantities in RL Parallel Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

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OVERVIEW

LESSON I

Solving for Quantities in RL Parallel Circuits

In this lesson you will study and learn about the following:

- -review parallel resistive circuits
- -parallel resistive-inductive circuits
- -methods of solution of RL parallel circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON I

Solving for Quantities in RL Parallel Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current."
Fundamentals of Electronics. Bureau of Naval Personnel.
Washington, D.C.: U.S. Government Printing Office, 1965

AUDIO-VISUAL:

Slide-Sound Presentation - "Solving for I_T in a Parallel RL Circuit."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

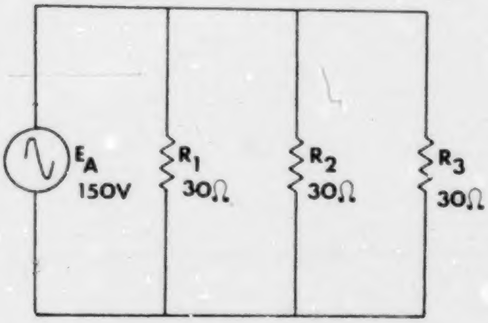
NARRATIVE

LESSON I

Solving for Quantities in RL Parallel Circuits

Reviewing Rules for Parallel Resistive Circuits

Several modules back, you learned to solve for quantities in parallel circuits which were purely resistive. Now we will briefly review the rules you used.



Rules

1. Branch resistance controls branch current.
2. Voltage is common across all branches.
3. Total resistance and total impedance must be less than smallest branch resistance or opposition.
4. Total current equals sum of branch currents.

Solve for the following quantities in the above circuit:

I_T = ____

Z_T = ____

E_{R2} = ____

I_{R1} = ____

I_{R2} = ____

I_{R3} = ____

P_t = ____

P_a = ____

$\underline{/\theta}$ = ____

PF = ____

To find I_T, add the branch currents. Voltage is common; therefore, the applied voltage is across R1, and we can determine $I_{R1} = \frac{150\text{ v}}{30\ \Omega}$ or 5 a. Similarly I_{R2} and I_{R3} are both 5 a.

5 a + 5 a + 5 a = 15 a (total current)

To solve for Z_T, you can use Ohm's Law.

$$Z_T = \frac{E_a}{I_T}$$

$$Z_T = \frac{150\text{ v}}{15\text{ a}}$$

$$Z_T = 10\ \Omega$$

Solving for E_{R2}

As voltage is common in a parallel circuit, the voltage drop across any of the branches is the same as the applied voltage, or 150 volts.

Solving for P_t

True power can be determined by the $P_t = I^2R$ formula. In this purely resistive circuit I = 15 a, $R_T = 10\ \Omega$; therefore, P_t = 2250 w or 2.25 kw.

Solving for P_a

Apparent power in a purely resistive circuit is equal to true power. You can prove this by the formula $P_a = E \times I$; P_a = 150 v x 15 a = 2.25 kw.

Solving for P_x

Reactive power is not present in a purely resistive circuit.

Angle Theta

In a purely resistive circuit, $\angle\theta$ is zero because E and I are in phase.

Power Factor

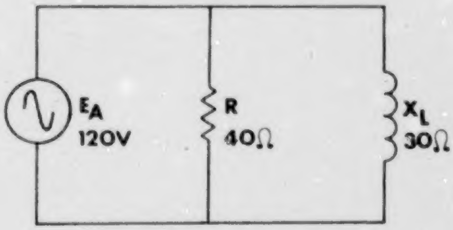
You know that the power factor in a purely resistive circuit is unity, or 1.

What would happen to total current in the parallel circuit we have been analyzing if frequency were doubled? ________________

You should know the answer is nothing. Current in parallel circuits is determined by branch resistance, and frequency usually does not affect the value of carbon (except for some skin effect at extremely high frequencies).

Parallel Resistive-Inductive Circuits

When solving purely resistive parallel circuits, branch currents are added directly because E and I are in phase. In circuits containing both resistive and reactive components, E and I are no longer in phase. In an RL parallel circuit, for example, I_T cannot be determined by directly adding branch currents because the current through the resistive branch is in phase with the applied voltage, while the current through the inductive branch lags the applied voltage by 90°.



To solve for I_T in this circuit, the sum of the branch currents has to be determined by vector addition.

Recall that in series AC circuits we had to solve for Z_T and voltage vectorially. We used the impedance triangle and the voltage triangle. In parallel AC circuits we will use only a current triangle. THINK CURRENT is the password for this module.

This means that we will not use an impedance triangle to solve for Z_T in parallel circuits. Instead we will first find I_T, then solve for Z_T by Ohm's Law.

In solving AC parallel circuits, a voltage triangle will never be used because voltage is common.

Which of these quantities would you not vector for when solving an AC parallel RL circuit?

Z_T ____________

I_T ____________

E_a ____________

Remember, for parallel circuits think current only; therefore, you do not vector for Z_T or E_a.

Solving the RL Parallel Circuit

We want to solve this circuit for each quantity listed to the left.

I_T = __________

Z_T = __________

I_R = __________

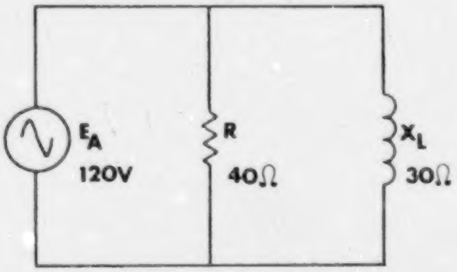
I_L = __________

P_t = __________

P_a = __________

$\underline{/\theta}$ = __________

PF = __________



First we must find current through each branch of the network in order to compute the total current.

To find I through the resistive branch:

$$I_R = \frac{E_a}{R}$$

$$I_R = \frac{120\text{ v}}{40\ \Omega} = 3\text{ a}$$

$$I_R = 3\text{ a}$$

Then, to find I through the inductor:

$$I_L = \frac{E_a}{X_L}$$

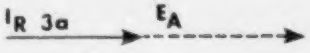
$$I_L = \frac{120\text{ v}}{30\ \Omega} = 4\text{ a}$$

$$I_L = 4\text{ a}$$

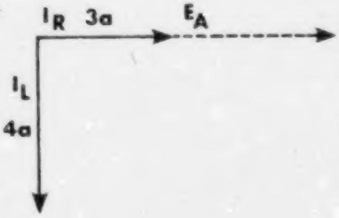
Because the 3 amps through the resistor and the 4 amps through the inductor are not in phase, we must add them vectorially.

The Current Triangle

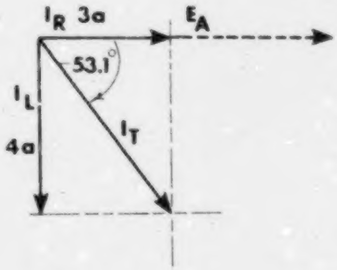
In parallel circuits, voltage is the common reference; therefore, voltage appears in the standard vector position. The voltage across



the resistor and the current through the resistor are in phase; therefore, I_R is plotted in the standard vector position.



Recall that in a purely inductive circuit, voltage leads current by an angle of 90° (ELI). For this reason E_a and I_L are 90° out of phase, with E_a leading; therefore, the I_L vector is a -j quantity. (Note that the -j is the result of changing the reference.)



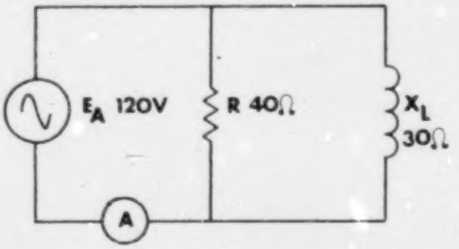
Vectorially adding the values of the branch currents produces the total circuit current. Therefore, I_T is 5 a.

This information can be expressed in rectangular notation, giving first the I through the resistor, then the I through the inductor, thus: 3 a - j4 a.

In polar form, this is expressed $5\ \underline{/-53.1^\circ}$. (You recall this is $\underline{/\theta}$ for a 3-4-5 triangle.)

Solving for Z_T

Now that we have found I_T, you can solve for total impedance by Ohm's Law. (Remember! We do not vector for Z_T in parallel circuits.)



$$Z_T = \frac{E_a}{I_T}$$

$$Z_T = \frac{120\ \underline{/0^\circ}}{5\ \underline{/-53.1^\circ}}$$ (E_a is our reference; therefore, we assign an angle of 0°.)

$$Z_T = 24\ \Omega\ \underline{/53.1^\circ}$$

Solving for True Power

You know that power is consumed only by resistance. To find the value of P_t, use the formula, $P_t = I^2R$. I in this case must be the current through the resistive branch, not total circuit current.

$P_t = I^2R$

$P_t = (3)^2 \times 40\ \Omega$

$P_t = 9 \times 40$

$P_t = 360$ w

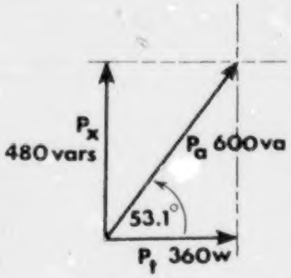
Solving for Reactive Power

$P_x = I^2_L X_L$

$P_x = (4)^2 \times 30$

$P_x = 16 \times 30$

$P_x = 480$ vars



Solving for Apparent Power

By the power formula $P = E_a \times I_T$

$P = 120$ v $\times$ 5 a

$P_a = 600$ va

Power factor

You know that the power factor is equal to the COS $\underline{/\theta}$. For $\underline{/\theta}$ of 53.1°, PF is 0.6. You can also determine this by using the formula $PF = \frac{P_t}{P_a}$.

Now, remember two important rules for solving parallel circuits:

1. THINK CURRENT - vector for $\underline{I}$ only.
2. To find Z_T, first find I_T, then apply by Ohm's Law.

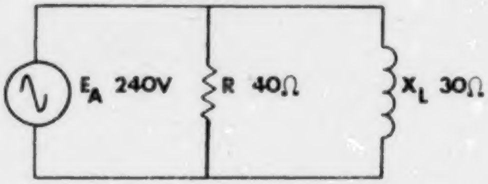
Solve this circuit.

I_R = ____________

I_L = ____________

I_T = ____________

Z_T = ____________



State I_T in rectangular form:

____

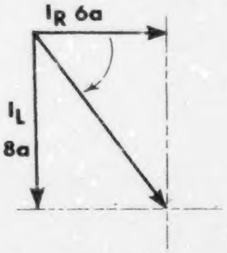
State I_T in polar form:

____

Your first step was to find I_R and I_L. $I_R = 6$ a; $I_L = 8$ a.

In rectangular form: $I_T = 6\text{ a} - j8\text{ a}$

The current triangle:



This is a 3-4-5 triangle, so $I_T = 10$ a

In polar form $I_T = 10\text{ a} \underline{/-53.1^\circ}$.

Then by Ohm's Law, $Z_T = \frac{E_a}{I_T}$,

$$Z_T = \frac{240\text{ v} \underline{/0^\circ}}{10\text{ a} \underline{/-53.1^\circ}} = 24\ \Omega \underline{/53.1^\circ}$$

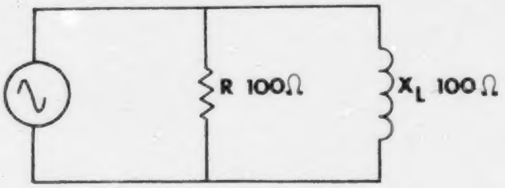
Observe that the two circuits we worked had identical ohmic values, 30 ohms and 40 ohms. Only the applied voltage was changed from 120 volts to 240 volts. Changing voltage did not affect the total impedance. In both circuits Z_T was 24 ohms. Voltage Does Not Affect Impedance in A Circuit.

Assumed Voltage Method

Since the applied voltage has no effect on Z_T, you may assume any voltage to solve for Z_T.

The assumed voltage (any one you pick that is easy to work with) will help you find the correct Z_T.

Using the assumed voltage method solve for Z_T.



$Z_T =$ __________

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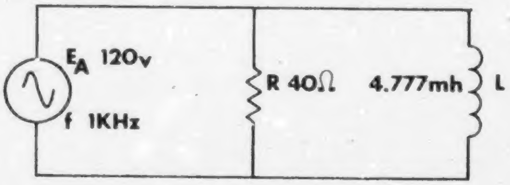
You may have realized a workable, easy, voltage to assume is 100 volts. $I_T = 1.41$ a $\underline{/-45^\circ}$, and $Z_T = 70.7\ \Omega\ \underline{/45^\circ}$. Whatever voltage you used, $Z_T = 70.7\ \Omega\ \underline{/45^\circ}$.

Parallel RL Problems

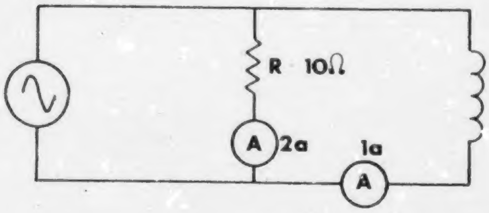
1.


E_A 120v
f 1KHz
R 40Ω
4.777mh
L

FIND

Z_T = ________

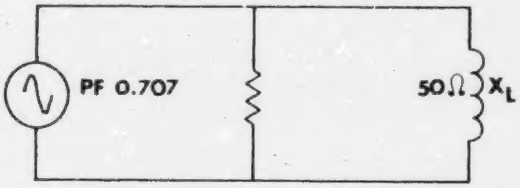
2.


R 10Ω
A
2a
1a
A

FIND

X_L = ________

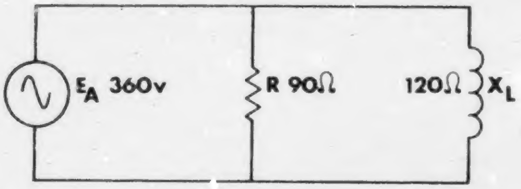
3.


PF 0.707
50Ω
X_L

FIND

Z_T = ________

4.


E_A 360v
R 90Ω
120Ω
X_L

FIND

Z_T = ________

I_T = ________

I_R = ________

I_L = ________

P_t = ________

P_a = ________

PF = ________

$\underline{/\theta}$ = ________

Answers

1. $Z_T = 24\ \Omega\ \underline{/53.1^\circ}$

2. $X_L = 20\ \Omega$

3. $Z_T = 35.35\ \Omega\ \underline{/45^\circ}$

4. $Z_T = 72\ \Omega\ \underline{/36.9^\circ}$

   $I_T = 5\text{ a }\underline{/-36.9^\circ}$

   $I_R = 4\text{ a}$

   $I_L = 3\text{ a}$

   $P_t = 1440\text{ w}$

   $P_a = 1800\text{ va}$

   $PF = 0.80$

   $\underline{/\theta} = -36.9^\circ$

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION

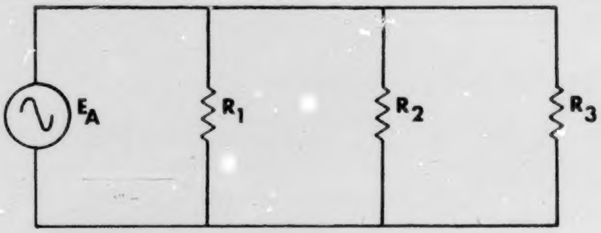
LESSON I

Solving for Quantities in RL Parallel Circuits

THIS PROGRAMMED SEQUENCE HAS NO TEST FRAMES.

In this module, we will discuss the important characteristics of parallel resistive-reactive circuits and develop the rules and graphs for finding circuit values. The basic circuit configurations to be studied include parallel RL, RC, and RLC types. At this time, it will be helpful to review parallel circuits containing only resistive elements.

1. In a series circuit, current is common, but in a parallel circuit, ________________ is common.



- -

(voltage)

2. The voltage drop across resistor R1 equals the voltage drop across R2 which equals the voltage drop across R3. This may be stated as "the voltage drop across any branch in a parallel circuit is equal to the ________________ voltage."

- -

(source or applied)

3. The current acts quite differently. The total current, I_T, flowing from the source divides into separate branch currents whose values are determined by the voltage and branch resistance. Given E_a and R1, we can calculate I_1 according to the equation

________________.

- -

$(I_1 = \frac{E_a}{R1})$

4. Equations for I_2 and I_3 are similarly written:

$$I_2 = \frac{E_a}{R2}$$

$$I_3 = \frac{E_a}{R3}$$

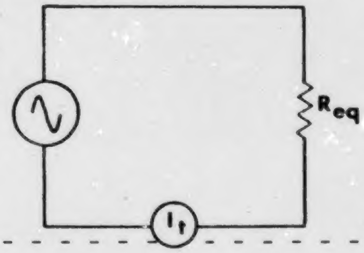
The sum of the branch currents must equal the ____________________ current.

(total)

5. For the circuit given in frame 1, the total current is written $I_T = I_1 + I_2 + I_3$. The total current can also be calculated from known values of source voltage and total resistance by the formula ____________________.

$$(I_T = \frac{E_a}{R_T})$$

6. The equivalent resistance of a parallel network can be calculated by the sum of the reciprocals method which is mathematically written as R_T = __________. The equivalent circuit for the parallel resistive circuit in frame 1 is:



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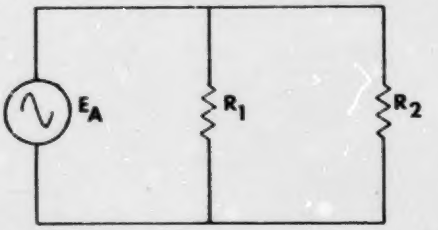
$$R_T = \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}}$$

7. For review, calculate the unknown values in the following examples.



E_a = 100 v

R1 = 5 Ω

R2 = 10 Ω

a. I_1 = _____

b. I_T = _____

c. R_T = _____

- -

(a. 20 a; b. 30 a; c. 3.3 Ω)

In summary, the rules for purely resistive parallel circuits are:

1. Voltage is common across each branch.
2. Total current is the sum of the individual branch currents:

   $I_T = I_1 + I_2 + . . . + I_n$

3. Total circuit current $I_T = \frac{E_a}{R_T}$ (Ohm's Law).
4. Branch currents are resolved using the equations $I_1 = \frac{E_a}{R_1}$, $I_2 = \frac{E_a}{R_2}$

   (where I is branch 1, 2, ... current.)

5. Total circuit resistance can be calculated from:

$$R_T = \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \ldots + \frac{1}{Rn}}$$

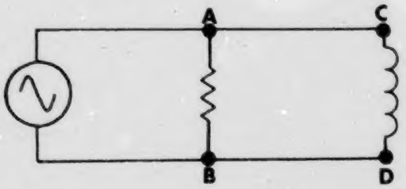
6. Phase difference between current and voltage is zero ($\underline{/\theta} = 0$).

7. True power equals apparent power since the load is purely resistive.

$$P_t = (I_T)^2 R_T = P_a = E_a I_T.$$

8. Power factor is unity: PF = 1.

8. The voltage drop between point A and B is ______________ to the drop across C and D



(equal)

9. In the circuit in the preceding frame, the voltage drops across the resistor, R, and the inductor, L, are the same and are equal to the ______________ voltage.

(source or applied)

10. From known values of source voltage, branch resistance, and inductive reactance, we can solve for the current through each branch of the network by ____________ Law.

(Ohm's)

11. Current flowing through the resistive branch in the example circuit, with $E_a = 100$ v and $R = 25\ \Omega$, is calculated from the equation $I_R =$ __________________ and is ___________ amps.

$\frac{E_a}{R}$, 4

12. We are able to calculate I_R from the equation $I_R = \frac{E_a}{R}$, since the voltage drop across the resistor is equal to __________.

(source or applied voltage)

13. Similarly, the voltage drop across the inductive branch equals source voltage. Branch current, I_L, can be calculated from the equation I_L = __________ and is __________ amps, with $E_a = 100$ v and $X_L = 20\ \Omega$.

$\frac{E_a}{X_L}$, 5

14. In a parallel RL circuit, total current, I_T, cannot be obtained by directly adding I_R and I_L as in a purely resistive circuit since the two currents are __________.

(out of phase)

15. With the voltage across each branch common and the currents in the resistive and inductive branches out of phase, we solve for total current by __________ addition of individual branch currents.

(vector)

16. In a parallel RL circuit, voltage is common and can be represented by a vector in the __________ position.

(standard or reference)

17. In the branch containing purely resistive elements, current flowing through R and the corresponding voltage drop, E_R, are in phase. Therefore, _______________ can be plotted in the same direction as E_a.

- -

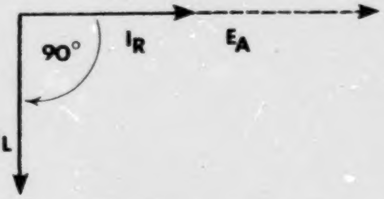
(I_R)

18. In the purely inductive branch of the parallel RL circuit, the same source voltage, E_a, appears across the inductance. The branch current, I_L, is out of phase with voltage. From "ELI, the ICE man," the voltage ____________ the current by 90°.

- -

(leads)

19. Voltage, however, is common and plotted in the standard position. Voltage leading current by 90° is the same as current __________ voltage by 90° for a purely inductive load.



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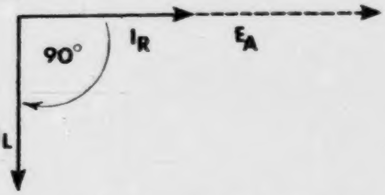
(lagging)

20. Current flow in a parallel RL circuit can be represented in rectangular notation with the lagging I_L current rotated clockwise from the standard position by a _____ operator. The
+j/-j
rectangular notation for the illustrated vector diagram is written _______________.



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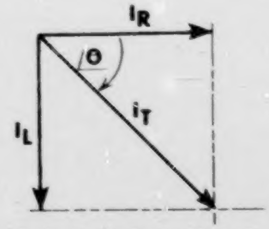
($-j$; $I_R - jI_L$)

21. The total current can now be calculated by vector addition of I_R and I_L. In the diagram, I_T is the ________ of the current vector triangle.



(resultant)

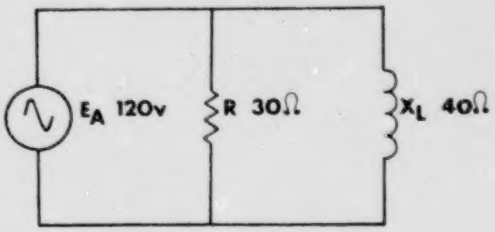
22. From the current vector diagram, you can see that I_T is out of phase and lags the applied voltage, E_a, by the ________.

($\underline{/\theta}$)

23. Total impedance, Z_T, can be calculated from known values of applied voltage and total current; from Ohm's Law, Z_T = ________.

$$\frac{E_a}{I_T}$$

24. In the example parallel RL circuit, the following values were calculated:



$I_R = 4$ a

$I_L = 3$ a

$I_T = 5$ a

$\underline{/\theta} = -36.9°$

Write I_T in rectangular form. ________________

Write total current in polar form. ________________

(4 a - j3 a; 5 a $\underline{/-36.9°}$)

25. In the preceding example, total impedance is determined from the formula, $Z_T = \frac{E_a}{I_T}$. We cannot simply divide 120 volts by 5 amps, but must also take into account the phase difference, $\underline{/\theta}$. Using polar notation, $Z_T = \frac{?}{5a\ \underline{/-36.9^\circ}}$.

- -

(120 v $\underline{/0^\circ}$; the phase angle for E_a is zero since it is the reference value.)

26. For example $Z_T = \frac{120v\ \underline{/0^\circ}}{5a\ \underline{/-36.9^\circ}}$, the total impedance is calculated to be Z_T = ________________.

- -

(24 Ω $\underline{/36.9^\circ}$

27. The total impedance of a parallel RL circuit is always calculated from applied voltage and total current, using Ohm's Law, and not by vector addition of resistance and impedance. In a fixed circuit with constant values of R and X_L, the total impedance, Z_T, does ________________ with changing values of E_a or I_T.
change/not change

- -

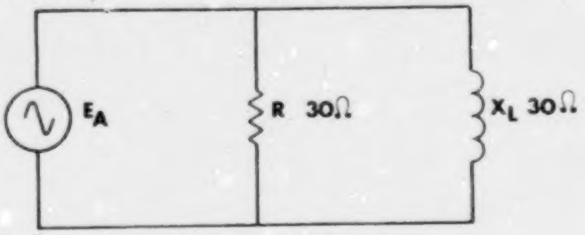
(not change)

28. From the equation, $E_a = I_T Z_T$, you can see that increasing E_a produces a proportional ________________ in I_T, since Z_T is constant for a given circuit.

- -

(increase)

29. In the circuit illustrated below, what happens to Z_T if E_a is increased from 120 volts to 240 volts?



____a. remains the same

____b. increases

____c. decreases

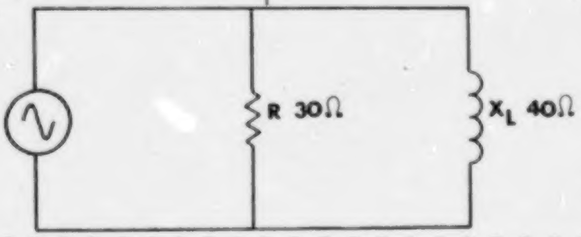
____d. cannot be determined

- -

(a)

30. This fact, that the applied voltage does not affect circuit impedance, allows us to assume any applied voltage to determine Z_T of a parallel circuit.

Assume E_a = 120 volts AC and solve for Z_T.



- -

(24 Ω $\angle 36.9°$)

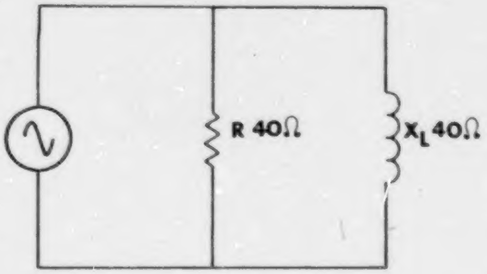
31. Solve the preceding circuit for Z_T, using an assumed voltage of 240 volts AC.

- -

(24 Ω $\angle 36.9°$)

32. When using the assumed voltage method for determining Z_T, the values of current are not true values.

Solve for I_T.



____a. 1 amp

____b. greater than 1 amp

____c. cannot be determined

- -

(c)

33. Recall that true power, P_t, is that power dissipated by the resistance as heat. In terms of I_R and R, P_t = ___________. For I_R expressed in amperes and R in ohms, P_t is expressed in ______________.

- -

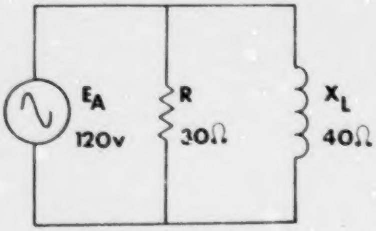
$(I_R)^2R$; watts)

34. True power may also be determined from the branch current and the voltage drop across the resistance using the equation P_t = __________________.

- -

($E_a \times I_R$)

35. Solve for true power using both equations and compare answers.
P_t = ____________ watts.



(480)

36. Recall also, that reactive power is that power stored by reactive components and returned to the source. In terms of I_L and X_L
P_x = ______________. For I_L expressed in amperes and X_L in ohms, P_x is expressed in ____________.

$(I_L)^2X_L$; vars

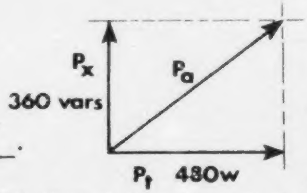
37. Reactive power may also be determined from the branch current and the voltage drop across the resistance using the equation P_x = _______.

$(E_a \times I_L)$

38. Using the circuit illustrated in frame 35, solve for reactive power. P_x = __________ vars.

(360)

39. Apparent power, P_a, is a combination of the power dissipated by the resistive components and the power stored by the reactive components.



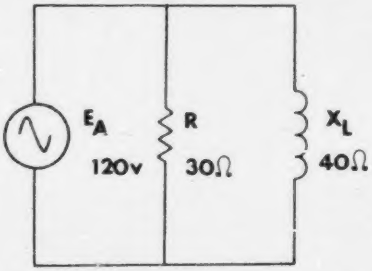
In terms of E_a and I_T, P_a = ____________.

- -

(E_a I_T)

40. When E_a is in volts and I_T in amperes, apparent power is expressed in ______________.

Solve for P_a.



P_a = ____________

- -

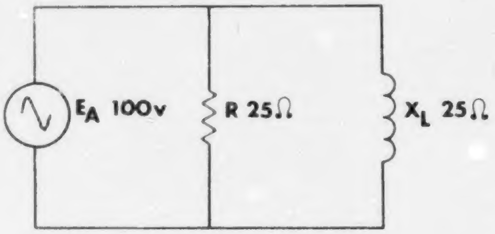
(volt-amperes; 600 va)

41. The power factor for any circuit is defined as the ratio of true power to __________ power. It may also be expressed as the ______________ of the $\angle\theta$.

- -

(apparent; cosine)

42. Solve this parallel RL circuit for the indicated values.



a. I_R = ______

b. I_L = ______

c. I_T = ______

d. Z_T = ______

e. $\underline{/\theta}$ = ______

f. P_t = ______

g. P_x = ______

h. P_a = ______

i. PF = ______

State total current in rectangular form. ______

State total current in polar form. ______

- -

(a. 4a; b. 4a; c. 5.7a $\underline{/-45°}$; d. 17.5 Ω $\underline{/45°}$; e. -45°; f. 400 w; g. 400 vars; h. 570 va; i. 0.7; rectangular form: I_T = 4 a - j4 a; polar form: I_T = 5.7 a $\underline{/-45°}$

YOU MAY NOW TAKE THE PROGRESS CHECK OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY

LESSON 1

Solving for Quantities in RL Parallel Circuits

In Module Six you were introduced to the rules governing parallel circuits. Before the discussion of the more complex AC circuit, perhaps a brief review of these rules, using a purely resistive circuit, will be helpful.

1. In a parallel network voltage is the common value.

   $E_a = E_1 = E_2 = E_3 = \ldots$

2. Total circuit current equals the sum of the branch currents.

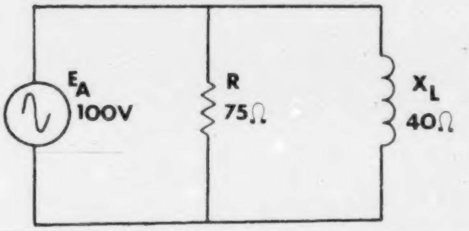
   $I_T = I_1 + I_2 + I_3 + \ldots + I_n$

3. Branch resistance determines branch current, with the branch containing the larger opposition having the smaller current.

4. Total circuit resistance is smaller than the smallest branch resistance.

When dealing with purely resistive circuits, we are able to add the branch currents directly to find total current; $\underline{E}$ and $\underline{I}$ are in phase, $\underline{/\theta}$ is 0°, and the circuit power factor is 1. If a reactive element (inductance in this case) is placed in the circuit, there is a phase shift between the applied voltage and circuit current. Because of this, other forms of computation must be used. For example, the individual branch currents can no longer be added algebraically to find I_T; vectorial addition must be used.

In series AC circuits, current is the common value and the individual voltage drops are added vectorially to find E_a. In parallel circuits, voltage is the common value and current divides between the branches. Hence, voltage, not current, must be used as the vector reference in a parallel circuit.

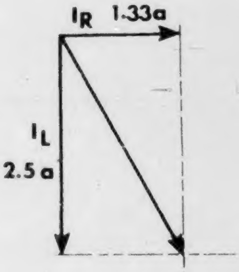


To solve this circuit with the information given, the first step is to find the individual branch currents. To do this, Ohm's Law is applied to each branch. (Remember to include the angle associated with each value.)

$$I_R = \frac{E}{R} = \frac{100\ \underline{/0^\circ}}{75\ \underline{/0^\circ}} = 1.33\text{a}\ \underline{/0^\circ} \qquad I_L = \frac{E}{X_L} = \frac{100\ \underline{/0^\circ}}{40\ \underline{/+90^\circ}} = 2.5\ \text{a}\ \underline{/-90^\circ}$$

E_a has a 0° angle because it is the reference value.

The next step is to find I_T. Since there is a 90° phase angle between the current through the resistive branch (I_R) and the current through the inductive branch (I_L), vector addition must be used to combine I_R and I_L.



$$I_T = I_R - jI_L$$

$$1.33\ \text{a} - j2.5\ \text{a} = 2.83\ \text{a}\ \underline{/-62^\circ}$$

Although this is an inductive circuit, the circuit phase angle is a negative angle. The reason for this is apparent; reversal is due to the change in reference values (current lags voltage).

Now that I_T has been computed, Z_T can be found by applying Ohm's Law.

$$Z_T = \frac{E_a}{I_T} = \frac{100\ \text{v}\ \underline{/0^\circ}}{2.83\ \text{a}\ \underline{/-62^\circ}} = 35.4\ \Omega\ \underline{/+62^\circ}$$

Remember, In Parallel Circuits You Must Vector For I_T, Not $\underline{Z}$ or E_a.

Apparent power is computed for parallel circuits in the same manner as for series. In the above example:

$$P_a = I_T \times E_a = (2.83)(100) = 283\ \text{va}$$

PF is still equal to the ratio of true power to apparent power.

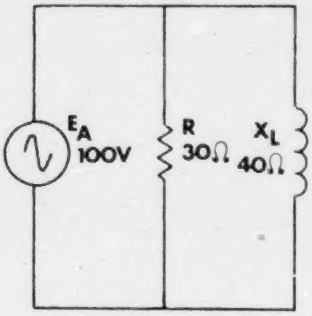
$PF = \frac{P_t}{P_a}$, or more conveniently, $PF = \text{COS}\ \underline{/\theta}$.

$$PF = \text{COS}\ 62^\circ = 0.4695$$

True power can be computed for any circuit by using $P_t = I_T \times E_a \times \text{COS}\ \underline{/\theta}$ or if you wish, $P_t = I_R^2 \times R$ or $P_t = I_R \times E_R$. The last equation is usually the most convenient for parallel circuits.

$$P_t = I_R \times E_R = (1.33)(100) = 133\ \text{w}$$

One fact you must always remember is that the impedance of a circuit is not affected by any change in applied voltage. This fact is apparent if you examine two identical circuits with different values of E_a.



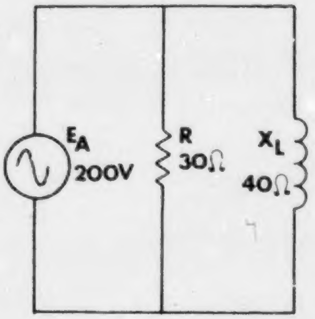
$I_R = 3.3$ a

$I_L = 2.5$ a

$I_T = 4.17$

$\underline{/\theta} = 36.9^\circ$

$Z_T = 24\ \Omega$



$I_R = 6.6$a

$I_L = 5$ a

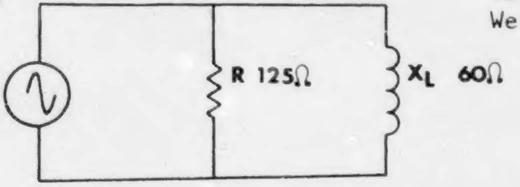
$I_T = 8.34$ a

$\underline{/\theta} = 36.9^\circ$

$Z_T = 24\ \Omega$

As you can see, the circuit phase angle does not change and Z_T remains the same.

Occasionally you will be required to find the impedance of a circuit when the value of the applied voltage is not known. Because the value of E_a has no effect on the impedance of the circuit, you can assume any voltage you wish to find Z_T. For example, any value of assumed voltage can be used for this circuit. We'll use 250 v.



$$I_R = \frac{250\ \underline{/0^\circ}}{125\ \underline{/0^\circ}} = 2\ \underline{/0^\circ}\ \text{a} \qquad I_L = \frac{250\ \underline{/0^\circ}}{60\ \underline{/+90^\circ}} = 4.16\ \text{a}\ \underline{/-90^\circ}$$

$$I_T = 2 - j4.16\ \text{a}$$

$$4.65\ \text{a}\ \underline{/-64.3^\circ}$$

$$Z_T = \frac{E}{I} = \frac{250\ \underline{/0^\circ}}{4.65\ \underline{/-64.3^\circ}} = 53.8\ \Omega\ \underline{/+64.3^\circ}$$

The circuit impedance found in this manner is a valid figure; however, you must keep in mind that I_T is correct only if the assumed E_a is applied.

In series circuits, the circuit is either predominately resistive or predominately reactive depending upon which device developes the largest voltage drop. A parallel circuit, however appears reactive or resistive depending on which branch carries largest current.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, SELECT ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-14a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON II

Variational Analysis of RL Parallel Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON II

Variational Analysis of RL Parallel Circuits

In this lesson you will study and learn about the following:

-effect of a change in frequency
-effect of a change in applied voltage
-effect of a change in resistance
-effect of a change in inductance

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON II

Variational Analysis of RL Parallel Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

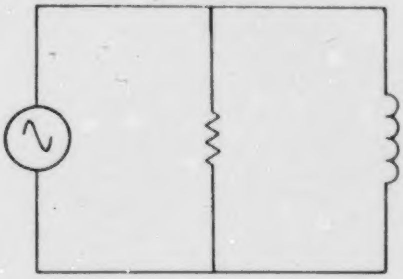
NAVPERS 93400A-1b "Basic Electricity, Alternating Current." Fundamentals of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE

LESSON II

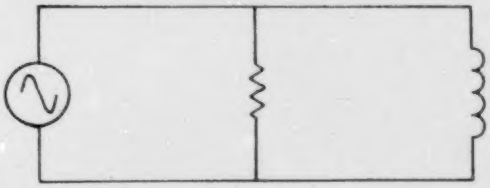
Variational Analysis of RL Parallel Circuits



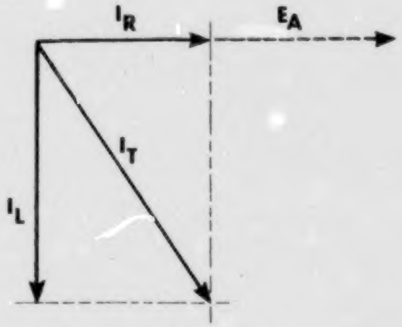
We will assume that this resistive-inductive parallel network has a high-Q coil; therefore, we will not need to consider effective resistance as we analyze the circuit.

We will examine the network to determine what will happen to all other circuit quantities if we change one of the four variables -- frequency, applied voltage, resistance, or inductance.

If Frequency Is Increased



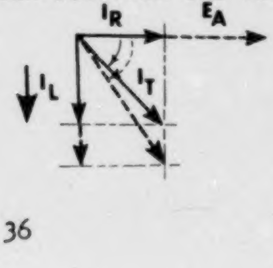
The first step in analyzing a parallel circuit is to draw the current vector diagram.



Now if f is increased, what will happen to X_L?

Since $X_L = 2\pi fL$, if f is increased X_L increases (↑).

If X_L increases, I_L decreases (↓). I_R in a parallel RL circuit is unchanged by a change in frequency because neither E_a nor R are affected by frequency. Therefore, if I_L decreases, then I_T decreases (↓) as we can see from the vector diagram.



If I_T decreases, Z_T must have increased (↑). We can see this by a look at Ohm's Law -- $Z_T \uparrow = \frac{E_{a\rightarrow}}{I_{T\downarrow}}$.

Since current through the resistive branch has not changed, and the resistance has not changed, true power does not change.

Apparent power, however, changes because total current decreases and the applied voltage is unchanged.

Looking again at the current vector diagram, you see that as I_L decreases, $\underline{/\theta}$ decreases.

If angle theta decreases, then the power factor increases. You know this to be true because P_t is the same, but P_a decreases.

$$PF\uparrow = \frac{P_{t\rightarrow}}{P_{a\downarrow}}$$

The table to the right shows the complete picture of the effects of an increase in the frequency of the voltage applied to a parallel RL circuit.

f	↑
Z_T	↑
I_T	↓
I_R	→
I_L	↓
X_L	↑
PF	↑
P_t	→
P_a	↓
P_x	↓
$\underline{/\theta}$	↓
R	→
L	→

Changing Applied Voltage

The results of a change in the voltage applied to a parallel FL circuit are easily analyzed.

None of the physical properties of the circuit are affected by any change in voltage; thus, R and L remain the same. Without a change in either frequency or inductance, X_L remains the same. With both X_L and R unaffected, the circuit impedance is also unaffected.

If E_a is decreased, Ohm's Law shows that I_L, and I_R and I_T also decreased.

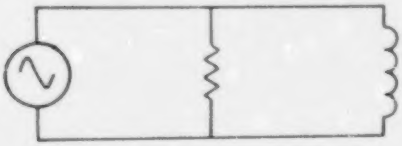
With a decrease in I_T, I_R and the applied voltage, P_t and P_a both decrease.

A look at the current vector diagram indicates that as I_R, I_L, and I_T all decrease, $\angle\theta$ does not change.

Therefore, PF does not change.

E_A	↓
Z_T	→
I_T	↓
I_R	↓
I_L	↓
X_L	→
PF	→
P_t	↓
P_a	↓
P_x	↓
$\angle\theta$	→
R	→
L	→

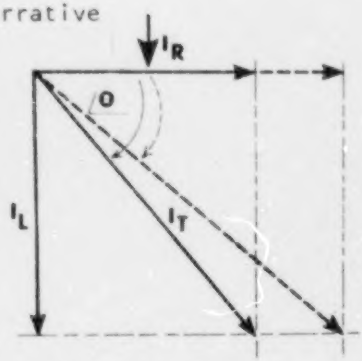
Okay, let's see if you can correctly analyze this circuit if the value of R is increased. Indicate changes to each circuit quantity by arrows in the table at the right.



Draw the current vector diagram.

R	↑
Z_T	
I_T	
I_R	
I_L	
X_L	
PF	
P_t	
P_a	
P_x	
$\angle\theta$	
R	
L	

You know that if R is increased, then I through the resistive branch decreases. I_L is unaffected since a change in R doesn't change either frequency or inductance; therefore, it does not affect X_L.



The vector diagram indicates that if I_R decreases and I_L does not change, I_T decreases.

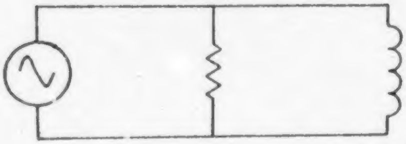
If I_T decreases, and E_a is constant, then Z_T increases.

If I_R and I_T decrease and E_a is unchanged, then P_t and P_a decrease.

Another look at the current vector diagram shows that $\angle\theta$ increases and the power factor decreases.

R	↑
Z_T	↑
I_T	↓
I_R	↓
I_L	→
X_L	→
PF	↓
P_t	↓
P_a	↓
P_x	→
$\angle\theta$	↑
R	↑
L	→

Complete the table to show what happens if inductance is decreased.

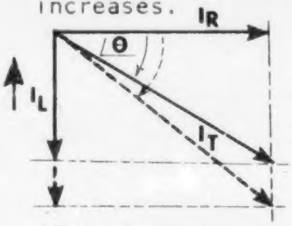


L	↓
Z_T	
I_T	
I_R	
I_L	
X_L	
PF	
P_t	
P_a	
P_x	
$\angle \theta$	
R	
L	

Turn to the next page for the answers.

Answers:

If L is decreased, X_L decreases, and I_L increases.



I_T increases.

I_R does not change.

If I_T increases and E_a is constant, Z_T decreases.

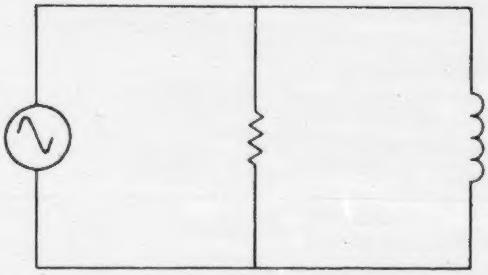
As neither I_R nor R changes, P_t does not change.

As I_T increases, P_a increases.

Obviously $\angle\theta$ increases; therefore, PF decreases, R remains the same.

L	↓
Z_T	↓
I_T	↑
I_R	→
I_L	↑
X_L	↓
PF	↓
P_t	→
P_a	↑
P_x	↑
$\angle\theta$	↑
R	→
L	↓

With the four variables changed separately as indicated in the chart, conduct a variational analysis of the circuit. Indicate the changes to each circuit quantity by arrows.



	f ↓	E_A ↑	R ↓	L ↑
Z_T				
I_T				
I_R				
I_L				
X_L				
PF				
P_t				
P_a				
P_x				
$\angle \theta$				
R				
L				

Turn to the next page for the answers.

Answers:

	f ↓	E_A ↑	R ↓	L ↑
Z_T	↓	→	↓	↑
I_T	↑	↑	↑	↓
I_R	→	↑	↑	→
I_L	↑	↑	→	↓
X_L	↓	→	→	↑
PF	↓	→	↑	↑
P_t	→	↑	↑	→
P_a	↑	↑	↑	↓
P_x	↑	↑	→	↓
$\angle\theta$	↑	→	↓	↓
R	→	→	↓	→
L	→	→	→	↑

PROGRAMMED INSTRUCTION

LESSON II

Variational Analysis of RL Parallel Circuits

In this lesson, we will perform a variational analysis on parallel RL circuits to determine what effect changes in frequency, source voltage, resistance, and inductance have on the circuit values. You remember from past lessons that a variational analysis of the circuit values consists basically of choosing one quantity as an independent variable and documenting the relative changes of all other values as the independent variable is changed.

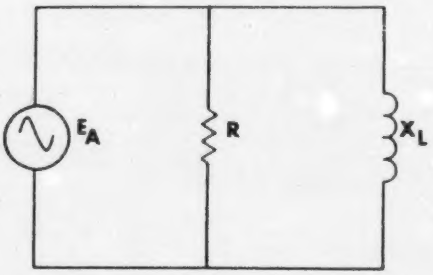
In the basic parallel RL circuit, we have the ability to vary or change four quantities. Each quantity, in turn, is considered the independent variable and a variational analysis performed. We will assume that the effective resistance of the coil is negligible, $R_{eff} = 0$.

The four independent variables are:

1. frequency
2. voltage
3. resistance
4. inductance

Before performing the analysis, it will be helpful to briefly review the characteristics of parallel RL circuits and present the fundamental equations used to calculate circuit values.

Basic Circuit



Rules and Equations

Voltage is common.

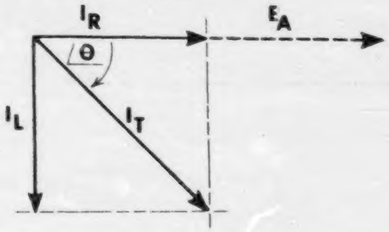
$$X_L = 2\pi f l$$

$$I_R = \frac{E_a}{R}$$

$$I_L = \frac{E_a}{X_L}$$

$I_T = I_R + I_L$ (vector sum)

Current Vector Diagram



The only vector diagram calculations are for current values.

$$\text{TAN } \underline{/\theta} = \frac{I_L}{I_R}$$

$$P_t = (I_R)^2R \text{ or } I_R \times E_a$$

$$P_a = E_a I_T$$

$$PF = \frac{P_t}{P_a} = \text{COS } \underline{/\theta}$$

Case I - Frequency of AC Voltage

f	L	R	X_L	Z_T	I_R	I_L	I_T	$\underline{/\theta}$	P_t	P_A	P_x	PF
↑												

We have chosen to increase the frequency of the AC source, keeping E_a, R and L constant. In the following frames, we discuss the effect of increasing f on the remaining circuit variables. In our circuit example, effective resistance of the inductor is assumed negligible with $R_{eff} = 0$.

THERE ARE NO TEST FRAMES IN THIS PROGRAMMED SEQUENCE.

---

1. Resistance is a physical property of the circuit. Increasing source frequency produces ________ change in R. How do you increase resistance? ____________________

(no; increase value of resistor)

2. Impedance of the inductor is given by the formula: $X_L = 2\pi fl$. An increase in f produces a/an __________ in X_L. X_L is measured in __________.

(increase; ohms)

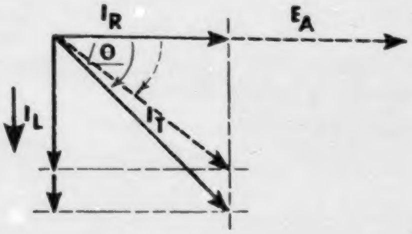
3. Increasing the frequency produces a corresponding increase in X_L. Branch current through X_L correspondingly ________________, according to Ohm's Law: $I_L =$ ____________.

(decreases; $\frac{E_a}{X_L}$)

4. Current flowing through the resistive branch depends only on E_a and ________________ according to Ohm's Law. Therefore, I_R __________________ when f is increased.

(R; does not change)

5. Knowing the changes to I_R and I_L, the dependence of total current (I_T) on increasing frequency can be determined from the current vector diagram. Increasing frequency causes a decrease in I_L and a/an __________ in I_T.



(decrease)

6. From the same current vector diagram, we can determine that the phase angle ____________.

(decreases)

7. Total circuit impedance (Z_T) can be expressed as a function of E_a and ____________ according to Ohm's Law: Z_T = ____________.

Increasing f causes Z_T to ________________.

- -

(I_T; $\frac{E_a}{I_T}$; increase)

8. The current flowing through the resistive branch does not change with an increase in frequency since resistance is a physical property. As shown by the formula, $I_R^2 \times R$, true power ______________________ with an increase in f.

- -

(does not change)

9. The current flowing through the inductive component decreases with an increase in frequency. As shown by the equation:

$X_L = 2\pi fL$, $I_L = \frac{E_a}{X_L}$ and $P_x = I^2 X_L$, reactive power _______________ with an increase in frequency.

- -

(decrease)

10. To find the variation in apparent power, consider the basic equation, $P_a = E_a I_T$.

Since I_T decreases with an increase in f, P_a __________________.

- -

(decreases)

11. Variations in the power factor can be determined from either one of the formulas:

$$PF = \frac{P_t}{P_a}$$

$$PF = \cos \underline{/\theta}$$

With an increase in f, true power remains constant and apparent power decreases. Since $\underline{/\theta}$ decreases, COS $\underline{/\theta}$ ________________ and PF ________________ for a corresponding increase in f.

- -

(increases; increases)

12. Fill in the blank blocks with arrows to indicate increase (↑), decrease (↓), or no change (→).

f	L	R	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↑												

f	L	R	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↑	→	→	↑	↑	→	↓	↓	↓	→	↓	↓	↑

Case II - Source Voltage

In this analysis, we increase E_a and determine the relationships of the following variables.

E_A	L	R	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↑												

13. Resistance is a physical property and does not change with an increase in E_a. Reactance depends only on f and L, so X_L is unchanged. The total circuit impedance (Z_T) is a combination of R and X_L and therefore ______________ with an increase in source voltage.

(does not change)

14. Branch currents, I_R and I_L, depend on the applied voltage and branch impedance.

$$I_R = \frac{E_a}{R}$$

$$I_L = \frac{E_a}{X_L}$$

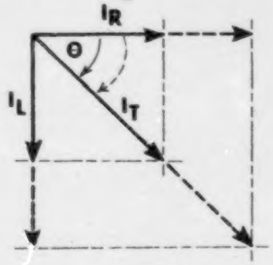
Therefore, since R and X_L are not affected by a change of applied voltage, I_R and I_L ________________ with an increase in source voltage, E_a.

(increase)

15. The dependence of total circuit current on source voltage can be determined from Ohm's Law; ($I_T = \frac{E_a}{Z_T}$). Increasing E_a produces a corresponding increase in ________________________.

(I_T)

16. In Ohm's Law, we can see that I_R and I_L increase a proportional amount for a given change in E_a. By looking at the current vector diagram, you can see that the phase angle ________________ with an increase in E_a.



(does not change)

17. True power, reactive power and apparent power can be expressed as the product of voltage and current as shown by the equations:

$$P_t = E_a I_R$$

$$P_x = E_a I_L$$

$$P_a = E_a I_T$$

As E_a is increased, I_R, I_L and I_T increase, producing corresponding ________________ in both power values.

(increases)

18. Since the power factor is equal to the cosine of /θ, a change in the applied voltage causes ____________ in PF.

(no change)

19. Place arrows in the blanks to show the change caused by an increase in source voltage.

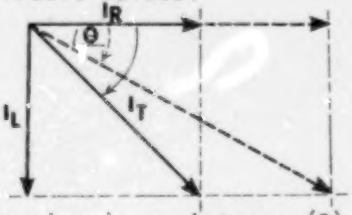
E_A	L	R	X_L	Z_T	I_R	I_L	I_T	/θ	P_t	P_A	P_x	PF
↑												

E_A	L	R	X_L	Z_T	I_R	I_L	I_T	/θ	P_t	P_A	P_x	PF
↑	→	→	→	→	↑	↑	↑	→	↑	↑	↑	→

Case III - Resistance

R	L	R	X_L	Z_T	I_R	I_L	I_T	$/\theta$	P_t	P_A	P_x	PF
↓												

In this example, we decrease the magnitude of the resistance. The following current vector diagram will help you visualize the changes in circuit values.



20. For a decrease in circuit resistance (R) determine the change, if any, to the following values.

a. E_a - Does a change in R cause E_a to change? __________

b. I_R - Current flowing through R ______________.

c. X_L - Since the reactance of the inductor depends on f and L, a decrease in R produces __________ in X_L.

d. I_L - ____________.

e. Z_T - Since total impedance is a combination of R and X_L, decreasing R causes ____________ in Z_T.

f. $/\theta$ - ______________.

g. P_t - I_R increases, producing ___________ in true power.

h. P_x - I_L does not change, producing __________ in P_x reactive power.

i. I_T - ______________.

j. P_a - ______________.

k. PF - The power factor equals COS $/\theta$. The phase angle decreases and PF ____________________.

- -

(a. no; b. increases; c. no change; d. no change;
e. a decrease; f. decreases; g. an increase; h. no change;
i. increases; j. increases; k. increases)

21. Show with arrows the effect of decreasing R.

R	L	E_A	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↓												

R	L	E_A	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↓	→	→	→	↓	↑	→	↑	↓	↑	↑	→	↑

Case IV - Inductance

22. In Case IV, we vary the inductance by decreasing L. For the remaining circuit variables, determine their dependence on decreasing L and indicate their variation by arrows in the following chart.

L	E_A	R	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↓												

L	E_A	R	X_L	Z_T	I_R	I_L	I_T	$\angle\theta$	P_t	P_A	P_x	PF
↓	→	→	↓	↓	→	↑	↑	↑	→	↑	↑	↓

YOU MAY NOW TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY

LESSON II

Variational Analysis of RL Parallel Circuits

During this lesson, we will examine a parallel network to determine what happens to circuit values when each of four variable quantities --- frequency, inductance, resistance, and applied voltage -- are changed. Remember, when dealing with parallel circuits, vector current, not impedance. The relative values of I_R and I_L determine whether the circuit is predominantly resistive or reactive.

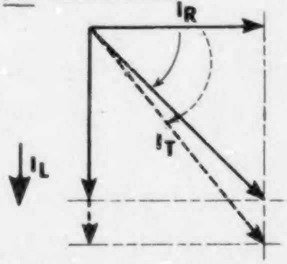
Just as in series circuits, the value most obviously affected by any change in frequency is inductive reactance. For example, an increase in frequency causes a corresponding increase in X_L.

An increase in the opposition of the inductive branch causes a decrease in the current through that branch.

The resistive branch, for practical purposes, is not affected by any variation in frequency.

Total current is equal to the vector sum of the resistive current (I_R) and the reactive current (I_L). In this case, with the decrease in I_L there is a corresponding decrease in I_T.

Since I_L decreases and I_R remains the same, the circuit appears more resistive and $\underline{/\theta}$ decreases.



With the decrease in $\underline{/\theta}$, there is an increase in PF.

Since total current decreases while the applied voltage remains the same, there must be an increase in impedance.

Although the PF increases, true power remains the same, because there is no change in the resistive branch.

Since apparent power is equal to total current times the applied voltage, it decreases.

To summarize:

f	↑
X_L	↑
R	→
I_L	↓
I_R	→
I_T	↓
Z	↑
$\angle\theta$	↓
PF	↑
P_t	→
P_a	↓
P_x	↓
L	→

Any variation in inductance affects the circuit in the same manner as a comparable change in frequency.

The table below shows the effect of a decrease in resistance in the resistive branch of an RL parallel circuit.

R	↓
Z_T	↓
I_T	↑
I_R	↑
I_L	→
X_L	→
PF	↑
P_t	↑
P_a	↑
P_x	→
$\angle\theta$	↓
R	↓
L	→

Any variation in applied voltage causes a proportional variation in all values of voltage, current, and power.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-14a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON III

Parallel RC and RCL Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON III

Parallel RC and RCL Circuits

In this lesson you will study and learn about the following:

- solving parallel RC circuits
- variational analysis of parallel RC circuits
- solving parallel RCL circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON III

Parallel RC and RCL Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

Lesson Narrative
Programmed Instruction
Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current." *Fundamentals of Electronics*. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

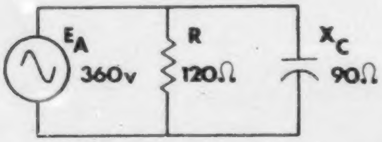
NARRATIVE

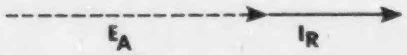
LESSON III

Parallel RC and RCL Circuits

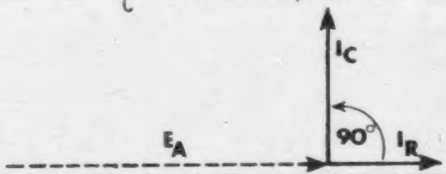
Solving Parallel RC Circuits

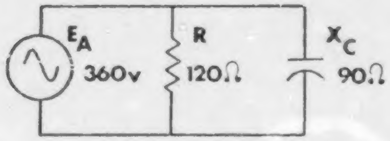
To solve for quantities in parallel RC circuits, you proceed just as you did to solve parallel RL circuits. Again with parallel RC circuits, we approach the problem via the vector diagram for current.





Here again, voltage is the common reference. Since current through the resistive leg is in phase with the applied voltage, I_R is vectored in the standard position. Recall that current through the capacitive branch leads the voltage by an angle of 90° (ICE). For this reason the vector for I_C is drawn upward to indicate I_C leading E_a.



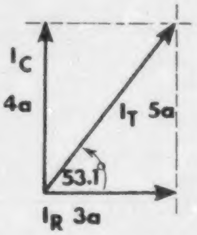


Find the values of I_R and I_C and construct the current vector diagram to determine I_T.

I_R = ____________ I_C = ____________

I_T = ____________

Your vector diagram should look like this:



I_R = 3 a

I_C = 4 a

I_T = 5 a $\underline{/53.1^\circ}$

By applying Ohm's Law, we can solve for Z_T.

$$Z_T = \frac{E_a}{I_T}$$

$$Z_T = \frac{360\text{ v }\underline{/0°}}{5\text{ a }\underline{/53.1°}}$$

$$Z_T = 72\ \Omega\ \underline{/-53.1°}$$

Solve the preceding circuit.

P_t ________________

P_a ________________

P_x ________________

True power is computed by either of the formulas: $P_t = (I_R)^2 \times R$, $P_t = I_R \times E_a$ or $P_t = I_T \times E_a \times \text{COS } \underline{/\theta}$

$$P_t = (I_R)^2 \times R$$

$$P_t = 9 \times 120$$

$$P_t = 1080\text{ w or }1.08\text{ kw}$$

Apparent power, is found using total circuit values.

$$P_a = I_T \times E_a = 5\text{ a} \times 360\text{ v}$$

$$P_a = 1800\text{ va}$$

Reactive power, is found by using total circuit values multiplied by the sine of $\underline{/\theta}$ ($\underline{/\theta} = -53.1°$) $P_x = I_T \times E_a \times \text{SIN } \underline{/\theta}$, or by using component values $P_x = I_C^2 \times X_C$. Once $\underline{/\theta}$ is known it is much simpler to use the sine function.

$$P_x = I_T \times E_a \times \text{SIN } \underline{/\theta}$$

$$P_x = 5\text{a} \times 360\text{v} \times .799$$

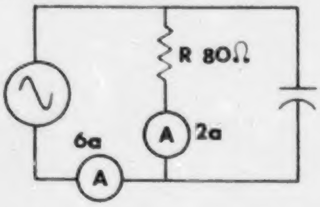
$$P_x = 1438\text{ vars}$$

Power factor is equal to the COS $\underline{/\theta}$ which is 0.6 for an angle of 53.1°.

The only difference between solving an RC parallel circuit and an RL parallel circuit is in the sign of the circuit phase angle. The phase angle is positive in RC circuits, but negative in RL circuits.

Practice Problems:

1.


R 80Ω
6a
A
A
2a

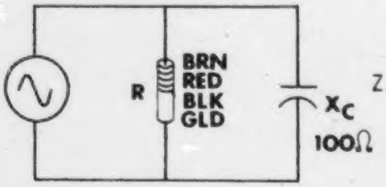
Z_T = ______

E_a = ______

X_C = ______

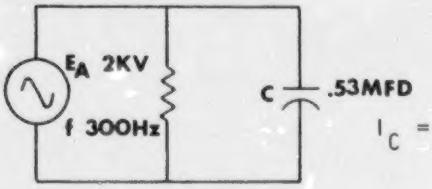
2. In problem 1 does the circuit appear more resistive or more capacitive?

3.


R
BRN
RED
BLK
GLD
X_C
100Ω

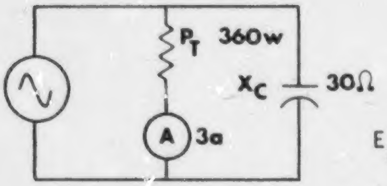
Z_T = ______

4.


E_A 2KV
f 300Hz
C
.53MFD

I_C = ______

5.


P_T 360w
X_C
30Ω
A
3a

E_a = ______

I_C = ______

I_T = ______

Z_T = ______

Answers:

1. $Z_T \approx 26.6\ \Omega\ \underline{/-70.5^\circ}$

   $E_a = 160$ v

   $X_C = 28.3\ \Omega\ \underline{/-90^\circ}$

2. Capacitive

3. $70.7\ \Omega\ \underline{/-45^\circ}$

4. 2 a

5. $E_a = 120$ v

$I_C = 4$ a $\underline{/90^\circ}$

$I_T = 5$ a $\underline{/53.1^\circ}$

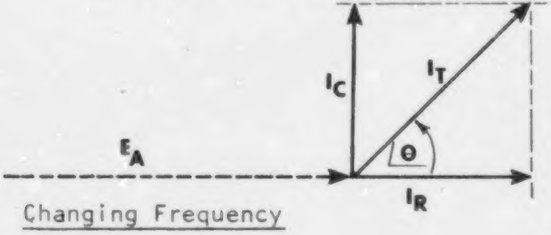
$Z_T = 24\ \Omega\ \underline{/-53.1^\circ}$

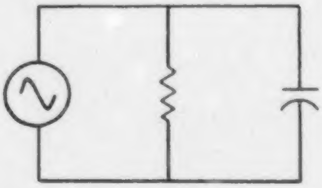
Variational Analysis of Parallel RC Circuits

When conducting a variational analysis of an RC circuit, keep in mind the formula

$$X_C = \frac{0.159}{fC}$$

and the current vector diagram.





Changing Frequency

If frequency is increased let's, see what happens to other circuit quantities.

If frequency is increased X_C decreases.

If X_C decreases, I_C increases.

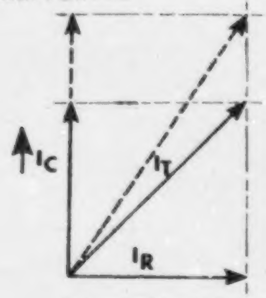
I_R remains the same because R is unaffected by a change in frequency.

If I_T increases and E_a remains the same, Z_T decreases.

If I_C increases, and I_R remains the same, I_T increases.

As I_T increases, P_a increases.

As I_C increases, P_x increases ($P_x = I^2_x\ (X_C)$)



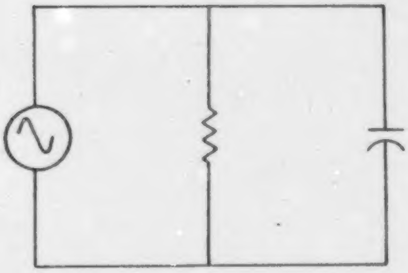
The current vector diagram shows that as I_C increases $\angle\theta$ increases.

If angle theta increases, PF decreases.

f	↑
Z_T	↓
I_T	↑
I_R	→
I_C	↑
X_C	↓
PF	↓
P_t	→
P_a	↑
P_x	↑
$\angle\theta$	↑
R	→
C	→

Capacitance is a physical property and is not affected by frequency.

Complete this variational analysis table using arrows to indicate the results of the changes indicated.

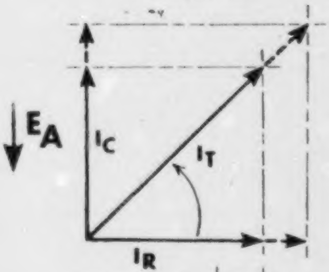


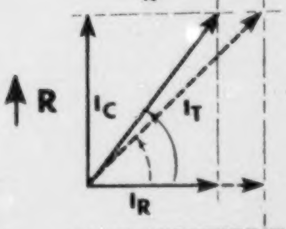
	E_A↓	R↑	C↓	f↓
Z_T				
I_T				
I_R				
I_C				
X_C				
PF				
P_t				
P_a				
P_x				
$\angle\theta$				
R				
C				

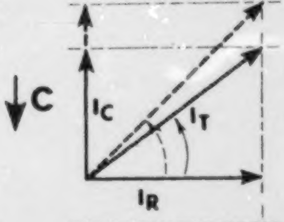
Draw the current vector diagrams.

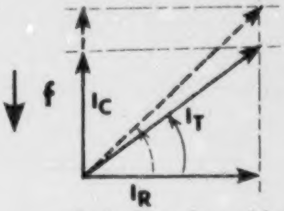
Check with answers on next page.

Answers:





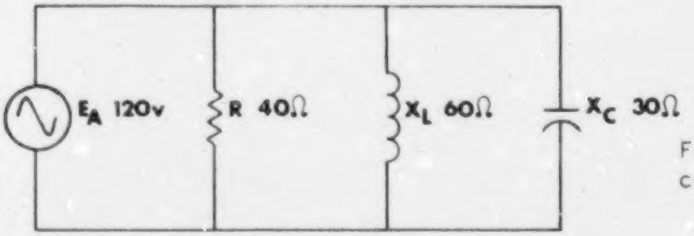




	E_A↓	R↑	C↓	f↓
Z_T	→	↑	↑	↑
I_T	↓	↓	↓	↓
I_R	↓	↓	→	→
I_C	↓	→	↓	↓
X_C	→	→	↑	↑
PF	→	↓	↓	↑
P_t	↓	↓	→	→
P_a	↓	↓	↓	↓
P_x	↓	→	↓	↓
$\angle\theta$	→	↑	↓	↓
R	→	↑	→	→
C	→	→	↓	→

Solving Parallel RCL Circuits

The objective here is to solve for each of the quantities listed at the right margin in this three-branch parallel circuit.



First by Ohm's Law, we solve for current through each branch and find that $I_R = 3$ a

$I_L = 2$ a

$I_C = 4$ a

I_R = ________

I_L = ________

I_C = ________

I_T = ________

Z_T = ________

P_t = ________

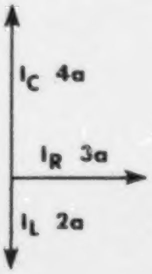
P_x = ________

P_a = ________

$\underline{/\theta}$ = ________

PF = ________

And we can draw these vectors.



In drawing the current vector diagram, we algebraically add the current quantities of I_C and I_L. This is possible because I_C and I_L are 180° out of phase.

Express in rectangular notation:

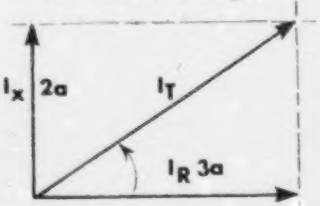
$I_R = 3\text{ a} + j0\text{ a}$

$I_L = 0\text{ a} - j2\text{ a}$

$I_C = \underline{0\text{ a} + j4\text{ a}}$

$I_T = 3\text{ a} + j2\text{ a}$

The vector diagram then shows $I_R = 3$ a and $I_X = 2$ a. This parallel RCL circuit appears to the source as a parallel RC circuit (more capacitive than inductive).



We may now convert I_T from rectangular notation to polar notation.

$$\text{TAN } \angle\theta = \frac{I_X}{I_R}$$

$$\text{TAN } \angle\theta = \frac{2\text{ a}}{3\text{ a}}$$

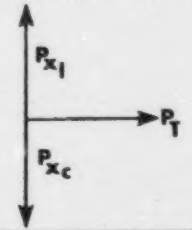
Therefore, $\text{TAN } \angle\theta = 0.6667$

Looking in the trig tables we find that when the TAN is 0.6667, then,

$\angle\theta = 33.7°$ $\text{COS } \angle\theta = 0.8320$

$\text{SIN } \angle\theta = 0.5548$ $I_T = 3.6 \text{ a } \angle 33.7°$

The combined values of inductive reactive power and capacitive reactive power is the circuit reactive power (P_X). Capacitive reactive power is 180° out of phase with the inductive reactive power.



Solve for Z_T of the preceding circuit.

Z_T = ________________

Solve for P_t, P_x and P_a.

P_t = ________________

P_a = ________________

P_x = ________________

Answers: $Z_T = 33.3\ \Omega\ \angle -33.7°$

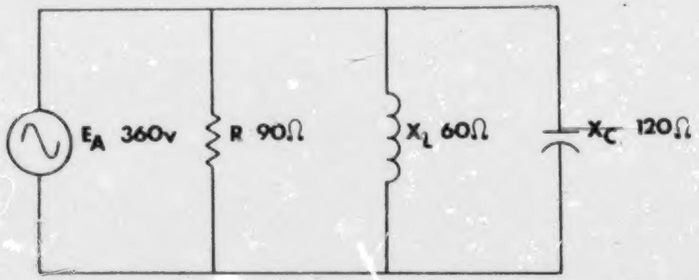
$P_t = 360$ w

$P_a = 432$ va

$P_x = 239.2$ vars

Practice

1.


E_A 360v
R 90Ω
X_L 60Ω
X_C 120Ω

Draw current vectors and solve.

I_T = __________

Z_T = __________

P_t = __________

P_x = __________

P_a = __________

$\underline{/\theta}$ = __________

PF = __________

I_R = __________

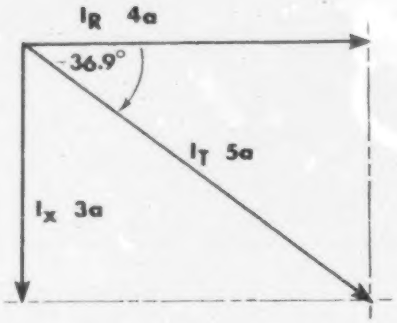
I_L = __________

I_C = __________

I_X = __________

Check answers on next page.

Answers:



$I_T = 5\ a\ \underline{/-36.9°}$

$Z_T = 72\Omega\ \underline{/36.9°}$

$P_t = 1440\ w$

$P_x = 1080\ vars$

$P_a = 1800\ va$

$\underline{/\theta} = -36.9°$

$PF = 0.8$

$I_R = 4\ a$

$I_L = 6\ a$

$I_C = 3\ a$

$I_X = 3\ a$

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

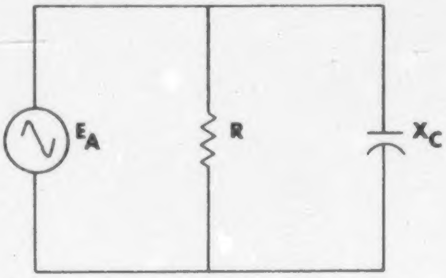
PROGRAMMED INSTRUCTION

LESSON III

Parallel RC and RCL Circuits

THIS PROGRAMMED SEQUENCE HAS NO TEST FRAMES.

1. A simple parallel RC circuit is illustrated by the diagram below. The only difference between this circuit and a parallel RL circuit is that a capacitor replaces the inductor. Consequently, voltage is still ______________, the voltage across the resistor equals the voltage across the capacitor, and both are equal to __________.



- -

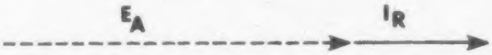
(common, E_a)

2. Since voltage is common, E_a is represented by a vector in the ____________________ position.

- -

(standard)

3. Current through the resistive branch is in phase with the ____________ and is represented by a vector in the standard position.



- -

(voltage)

4. The magnitude of I_R can be calculated by Ohm's Law.

I_R = ____________________

$\frac{E_a}{R}$

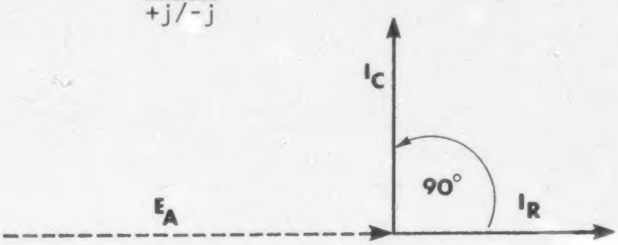
5. In parallel RC or RL circuits, the circuit value to vector for is ____________________.

(current)

6. In the purely capacitive branch, current ________________ voltage by 90°.

(leads)

7. In the current vector diagram, I_C is drawn as a vector pointing in the _____ position.
+j/-j

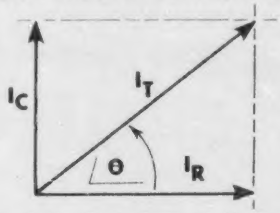


(+j)

8. Using the symbols I_R and I_C, the total current in the circuit can be written in rectangular notation as ____________________.

($I_R + jI_C$)

9. In the current vector diagram, the resultant vector represents the ________________ current flowing in the circuit and $\angle\theta$ is the angle between circuit current and ________________.

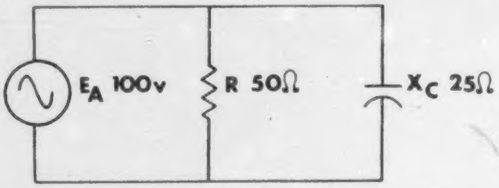


(total; voltage)

10. For the example circuit, I_R and I_C can be calculated according to Ohm's Law. The magnitude of branch currents are:

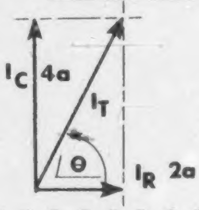
I_R = ________________

I_C = ________________



(2 a; 4 a)

11. The current vector diagram for the above example is given below. Total current can be calculated to be ________________ with a phase angle of ________________.



(4.5 a; 63.4°)

12. An important rule to remember in calculating total impedance was given in the discussion of parallel RL circuits. Namely, total impedance is calculated according to Ohm's Law for total current and source voltage. The equation for Z_T is written:

Z_T = ________________

- -

$\frac{E_a}{I_T}$

13. Now, the magnitude of the total impedance can be directly calculated to be ____________. To determine the phase of the total impedance, we must include the phases of E_a and I_T in the equation.

- -

($22\ \Omega\ \underline{/-63.4^\circ}$)

14. The source voltage (E_a = 100 v) is the reference value and can be written in polar form as:

E_a = __________

The total current written in polar form is:

I_T = __________

- -

($100\ v\ \underline{/0^\circ}$; $4.5\ a\ \underline{/63.4^\circ}$)

15. Using the equation, $Z_T = \frac{E_a}{I_T}$, the total impedance written in polar notation becomes ____________.

- -

($22\ \Omega\ \underline{/-63.4^\circ}$)

16. The true power of any circuit is that power dissipated in the resistive elements as heat. In parallel RC circuits, the equation for true power may be written in terms of I_R and R.

P_t = ____________

- -

(I_R^2R)

17. The true power for the example circuit is ____________.

For I_R and R given in amperes and ohms, respectively, the true power is expressed in ________.

- -

(200 w; watts)

18. Apparent power is equal to the product of source voltage and ________. The magnitude of P_a for the example circuit is ________ va.

- -

(total current; 450)

19. Reactive power is equal to the product of the source voltage, total current and the sine $\angle\theta$. The magnitude of P_x for the same circuit is ______ vars.

- -

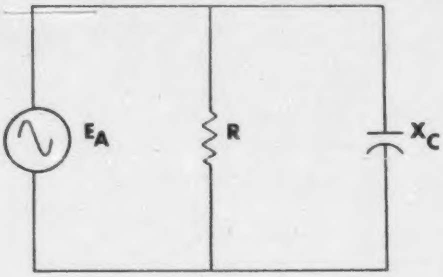
(402.3 vars)

20. In the variational analysis of parallel RC circuits, we independently vary the following circuit quantities and determine what happens to the remaining circuit variables.

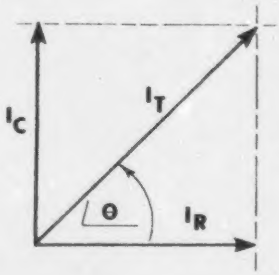
1. frequency (f)
2. source voltage (E_a)
3. capacitance (C)
4. resistance (R)

Again, it will be helpful to summarize the rules and equations describing parallel RC circuits. Frequent use of the current vector diagram will be made to visualize circuit characteristics.

Basic Circuit



Current Vector Diagram



Rules and Equations

Voltage is common

$$X_C = \frac{0.159}{fC}$$

$$I_R = \frac{E_a}{R}$$

$$I_C = \frac{E_a}{X_C}$$

$$I_T = I_R + jI_C$$

$$\text{TAN } \underline{/\theta} = \frac{I_C}{I_R}$$

$$P_t = (I_R)^2 R \text{ or } I_R E_a$$

$$P_x = I_x^2 \times X_C \text{ or } I_T E_a \text{ SIN } \underline{/\theta}$$

$$P_a = I_T E_a$$

$$PF = \frac{P_t}{P_a} = \text{COS } \underline{/\theta}$$

- -

21.

f	R	X_C	I_R	I_C	I_T	Z_T	$\underline{/\theta}$	P_T	P_x	P_A	PF
↑											

In this example, we increase frequency and determine the corresponding changes, if any, in the remaining circuit quantities. Upon completing the short statements, note the variation in the above table by appropriately directed arrows. Draw the current vector diagram when necessary.

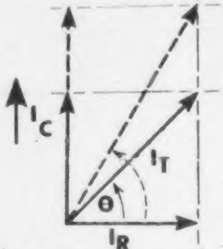
R - Resistance is a physical property; therefore R ____________ with an increase in frequency.

X_C - X_C is proportional to $\frac{1}{2\pi fC}$; therefore X_C ______________ with an increase in f.

I_R - Since R and E_a are constant, I_R _______________.

I_C - From Ohm's Law, I_C is expected to ____________ with a decrease in X_C.

I_T - With I_R constant and I_C increasing, I_T ________________.



$\underline{/\theta}$ - The vector diagram shows that the phase angle, θ, ____________.

Z_T - From Ohm's Law, $Z_T = \frac{E_a}{I_T}$, the total impedance ____________ with E_a held constant and I_T increased.

P_t - Since $\underline{R}$ and I_R are constant, P_t ______________.

P_a - Apparent power equals $E_a \times I_T$, and it __________ with an increase in $\underline{f}$.

PF - From the ratio of $\frac{P_t}{P_a}$, the power factor ___________.

- -

The completed matrix for a change in frequency is:

f	R	X_C	I_R	I_C	I_T	Z_T	$\underline{/\theta}$	P_T	P_x	P_A	PF
↑	→	↓	→	↑	↑	↓	↑	→	↑	↑	↓

Answers:

R - does not change; X_C - decreases; I_R - is also constant;

I_C - increase; I_T - increases; $\underline{/\theta}$ - increases; Z_T - decreases;

P_t - is constant; P_x - increases; P_a - increases; PF - decreases

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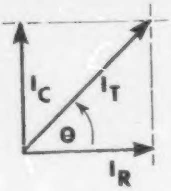
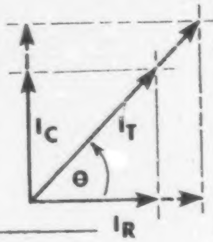
Case II - Source Voltage

22. Complete this table.

E_A	R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↑											


I_C
I_T
θ
I_R

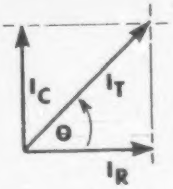
E_A	R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↑	→	→	↑	↑	↑	→	→	↑	↑	↑	→


I_C
I_T
θ
I_R

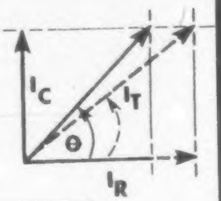
Case III - Resistance

23. Complete this table.

R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↓										


I_C
I_T
θ
I_R

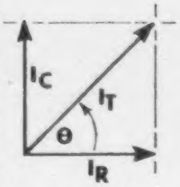
R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↓	→	↑	→	↑	↓	↓	↑	→	↑	↑


I_C
I_T
θ
I_R

Case IV - Capacitance

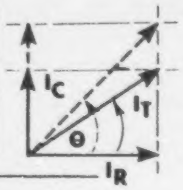
24. Complete this table.

C	R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↓											



- -

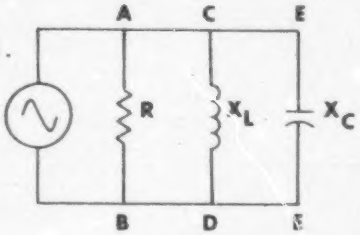
C	R	X_C	I_R	I_C	I_T	Z_T	$\angle\theta$	P_T	P_x	P_A	PF
↓	→	↑	→	↓	↓	↓	↓	→	↓	↓	↑



Parallel RCL Circuits

A simple parallel RCL circuit consists of one purely resistive branch, one purely inductive branch, and one purely capacitive branch. The basic rules for solving parallel RL and parallel RC circuits apply to a parallel RCL circuit.

25.



A basic RCL circuit is illustrated at the left. Voltage is ____________ as in the parallel RL and RC circuits. The total source voltage, E_a, appears across branches AB, CD, and __________.

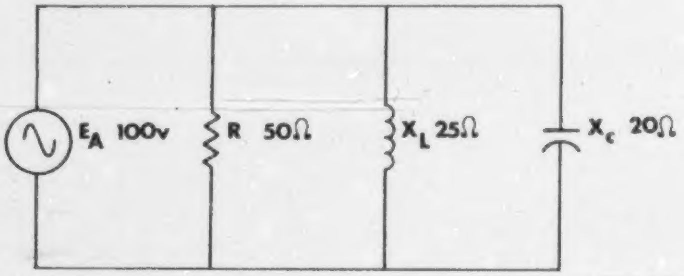
- -

(common; EF)

26. Given circuit values, we can directly solve for the current flowing through each of the three branches using __________ __________.

(Ohm's Law)

27. Solve for branch currents.



I_R = __________

I_L = __________

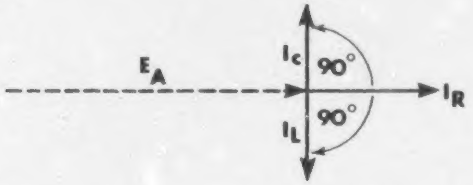
I_C = __________

(2 a; 4 a; 5 a)

28. E_a and the current flowing through the __________ branch are in phase; therefore, the vector representing I_R is plotted in the standard position.

(resistive)

29. Current in the inductive branch __________ the voltage by 90° while current in the capacitive branch __________ the voltage by 90°.



(lags; leads)

30. We cannot solve the current vector diagram yet since the circuit vectors point in three separate directions. First, we must represent branch currents in rectangular notation. In rectangular notation, the branch currents for the example circuit are:

I_R = ____________

I_L = ____________

I_C = ____________

- -

(2 a + j0 a; 0 a - j4 a; 0 a + j5 a)

31. We can add branch currents when they are expressed in rectangular notation. The resulting sum is:

I_R = 2 a + j0 a

I_L = 0 a - j4 a

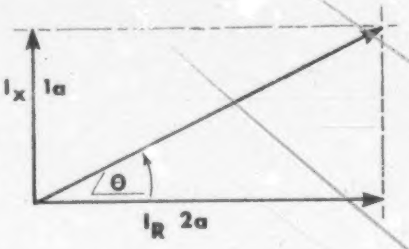
I_X = 0 a + j5 a

I_T = ____________

- -

(2 a + j1 a)

. We can now draw a vector diagram of the branch currents representing I_R and the difference between ______________ and ________________.



- -

(I_C; I_L)

33. Since all branch currents are represented in the vector diagram, we can solve for the ____________ circuit current.

- -

(total)

34. Convert I_T from rectangular notation to polar notation:

$I_T =$ ____________

- -

($2.24\text{ a }\angle 26.6^\circ$)

35. Knowing both the source voltage and total current, we can solve for the total impedance of the circuit using Ohm's Law.

$Z_T =$ __________

- -

$$\frac{100\text{v }\angle 0^\circ}{2.24\ \angle 26.6^\circ} = 44.8\ \Omega\ \angle -26.6^\circ$$

36. Determine true power, apparent power and reactive power for the example.

$P_t =$ __________

$P_a =$ __________

$P_x =$ __________

- -

(200 w; 224 va; 100 vars)

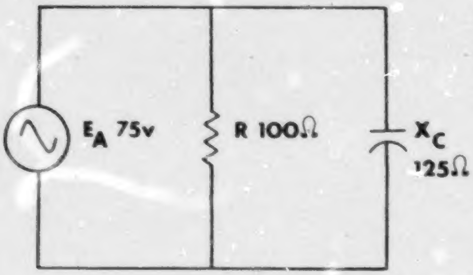
YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO ON TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY

LESSON III

Parallel RC and RCL Circuits

The procedures for solving parallel RC circuits are identical to those used to compute RL parallel values. The only difference is the sign of the resulting circuit phase angle. An inductive parallel circuit has a negative phase angle while a capacitive parallel circuit has a positive phase angle.

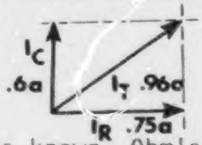


The first step is to apply Ohm's Law to each branch and find I_R and I_C.

$$I_R = \frac{75 \angle 0^\circ}{100 \angle 0^\circ} = \underline{0.75\text{ a} \angle 0^\circ}$$

$$I_C = \frac{75 \angle 0^\circ}{125 \angle -90^\circ} = \underline{0.6\text{ a} \angle +90^\circ}$$

Branch currents are added vectorially to find I_T and $\angle\theta$.



$$I_T = 0.75 + j0.6 = \underline{0.96\text{ a} \angle 38.6^\circ}$$

Once I_T is known, Ohm's Law is applied to compute Z.

$$Z = \frac{E_a}{I_T} = \frac{75 \angle 0^\circ}{0.96 \angle +38.6^\circ} = 78.2\ \Omega \angle -38.6^\circ$$

Power can now be determined:

$$P_t = (I_R)^2 \times R = (0.75^2)(100) = (0.56)(100) = 56\text{w}$$

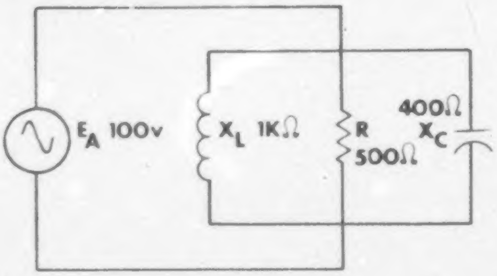
$$P_x = (I_C)^2 \times X_C = (0.6^2)\ (125) = (0.36)\ (125) = 45\text{ vars}$$

$$P_a = I_T \times E_a = (0.96)(75) = 72\text{ va}$$

$$PF = \frac{P_t}{P_a} = \frac{56}{72} = 0.78$$

Parallel RC circuits react to changes in voltage, impedance, or reactance the same way as parallel RL circuits. Keep in mind that X_C is inversely proportional to frequency and capacitance while X_L is directly proportional to frequency and inductance. An increase in frequency causes a decrease in capacitive reactance.

If you have successfully mastered RL and RC circuits, you will find that the three-branch RCL circuits will present no problem.



The first step is to find the current through each branch.

$$I_L = \frac{E_a}{X_L} = \frac{100 \angle 0^\circ}{1 \times 10^3 \angle +90^\circ} = 0.1 \text{ a} \angle -90^\circ$$

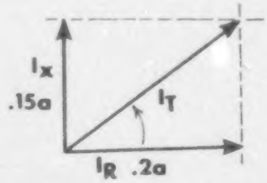
$$I_R = \frac{E_a}{R} = \frac{100 \angle 0^\circ}{500 \angle 0^\circ} = 0.2 \text{ a}$$

$$I_C = \frac{E_a}{X_C} = \frac{100 \angle 0^\circ}{400 \angle -90^\circ} = 0.25 \text{ a} \angle +90^\circ$$

Combine the branch currents using rectangular notation.

$$\begin{aligned} I_L &= 0 - j0.1 \\ I_R &= 0.2 + j0 \\ I_C &= 0 + j0.25 \\ \hline I_T &= 0.2 + j0.15 \end{aligned}$$

I_L and I_C tend to cancel each other and the result is 0.15 amps of reactive current. To find I_T, the resistive current (I_R) and the difference between I_L and I_C are added vectorially.



$0.25 \text{ a} \angle +36.8^\circ$

Once I_T has been determined, the other circuit values can be computed in exactly the same manner as in any other circuit. Because the mathematical procedure to derive reactance in the parallel RCL circuits is too lengthy, the sine function of the circuit phase angle is used to resolve the circuit value of P_x.

$$Z_T = \frac{E_a}{I} = \frac{100 \text{ v}}{0.25 \angle +36.8^\circ} = 400\ \Omega \angle -36.8^\circ$$

$$P_t = (I_R)^2 \times R = (0.04)(500) = 20 \text{ w}$$

$$P_x = I_T \times E_a \times \text{SIN} \angle \theta = (0.25)\ (100)\ (0.599) = 14.9 \text{ vars}$$

$$P_a = I_T \times E_a = (0.25)(100) = 25 \text{ va}$$

$$\text{PF} = \text{COS} \angle \theta\text{:}\quad \text{COS} \angle 36.8^\circ = 0.8007$$

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-14a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON IV

Resonant Frequency in Parallel AC Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

84/85

OVERVIEW

LESSON IV

Resonant Frequency in Parallel AC Circuits

In this lesson, you will study and learn about the following:

- analyzing a parallel LC circuit
- an ideal parallel LC circuit operating at its natural resonant frequency
- analyzing a practical tank circuit
- effects of operating parallel LC circuits at other than resonant frequency
- filters using parallel resonant circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON IV

Resonant Frequency in Parallel AC Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b "Basic Electricity, Alternating Current."
Fundamentals of Electronics. Bureau of Naval Personnel.
Washington, D.C.: U.S. Government Printing Office, 1965.

AUDIO-VISUAL:

Slide/Sound Presentation - "Resonance in Parallel Circuits."

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

NARRATIVE

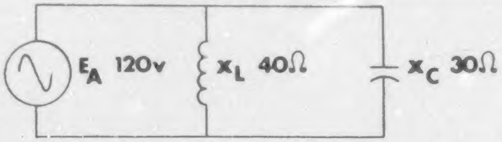
LESSON IV

Resonant Frequency in Parallel AC Circuits

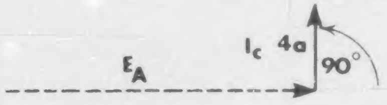
Analyzing an LC Circuit

So far in this module, we have analyzed parallel RL, RC and RLC circuits. Now we will idealize (disregard all circuit resistance) a circuit that has only reactive components, a capacitor and a coil.

Our task in analyzing this LC parallel circuit is to determine the total current and then total impedance.

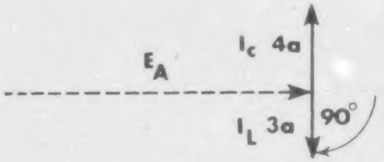


As this is a parallel circuit, we will analyze the problem using the current vector diagram. Again our common reference is the applied voltage.



Using Ohm's Law, we determine the current through the capacitive branch is 4 amps, and construct a vector for I_C to show that current leads the voltage by 90°.

We can determine that the current through the inductive branch is 3 amps. The vector for I_L is drawn to show that voltage leads current by 90°.



Because I_C and I_L are 180° out of phase, we can algebraically add these quantities to solve for total current.

$$I_C = 0 + j4 \text{ a}$$

$$I_L = \underline{0 - j3 \text{ a}}$$

$$I_T = 0 + j1 \text{ a}$$

Now, by using Ohm's Law, we can solve for Z_T.

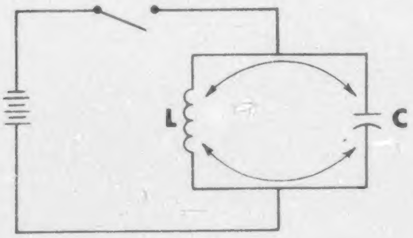
$$Z_T = \frac{E_a}{I_T}$$

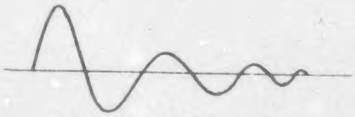
$$Z_T = \frac{120v}{1\ a}$$

$$Z_T = 120\ \Omega$$

Note that Z_T is greater than either X_C or X_L. This is an important aspect of a parallel LC circuit and we will discuss it further.

When the source of power is removed, current will continue to circulate betwen L and C.



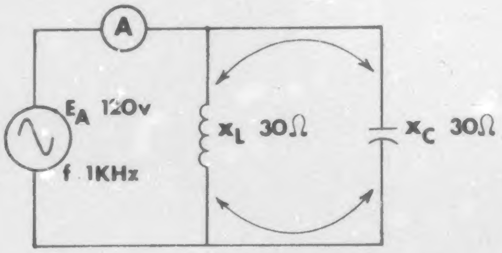


DAMPED WAVE

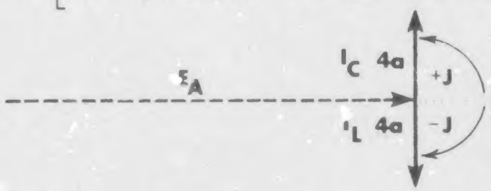
This action is due to the discharging of the capacitor and the collapsing field of the inductor. The current will continue to circulate back and forth between L and C at a diminishing rate. A damped wave is formed due to the resistance present in any practical circuit. This action is known as flywheel action or flywheel effect.

An Ideal Circuit at Natural Resonant Frequency

Notice that X_C and X_L are equal, both 30 ohms.



Current is flowing in both branches of the network. $I_C = +j4$ amps and $I_L = -j4$ amps. The current vector diagram is plotted below:



Because I_C and I_L are equal and opposite, they cancel each other; thus, the total circuit current is 0 since at any given instant of time, current is flowing in one direction through the capacitive branch of the circuit and in the opposite direction through the inductive branch. The appearance from the center point, C, is that of a loop of current. This is called circulating current or tank current, and the parallel LC circuit is called a tank circuit.

The point to be stressed is that there is no current drawn from the source. The line current equals 0. A high current flows in the tank, but no current flows to or from the source.

Solving for Z_T in the ideal circuit yields:

$$Z_T = \frac{120\text{v}}{0\text{ a}} = \text{infinity } (\propto)$$

(Dividing zero into any number actually yields an undefined number. However, if I_{LINE} becomes smaller and smaller, eventually there is no current flow from the source, and an infinite amount of opposition is implied.)

When these conditions exist:

$$X_C = X_L$$

$$I_{LINE} = 0$$

$$Z_T = \propto$$

The parallel circuit is said to be at its natural resonant frequency.

Natural resonance always occurs when $X_L = X_C$.

This is true for both series and parallel circuits.

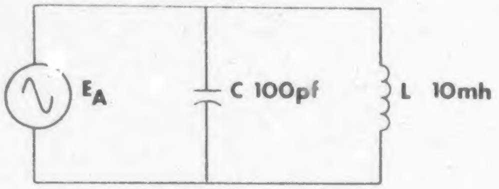
In practice, an "ideal circuit" never exists because resistance is always present. However, we will further discuss this ideal circuit to better understand what takes place at natural resonant frequency.

Recall that to find resonant frequency in series AC circuits, we used this formula which came from the $X_L = X_C$ condition:

$$f_o = \frac{0.159}{\sqrt{LC}}$$

Since $X_L = X_C$ in parallel circuits at resonance also, the same formula is used to find f_o.

Find f_o.



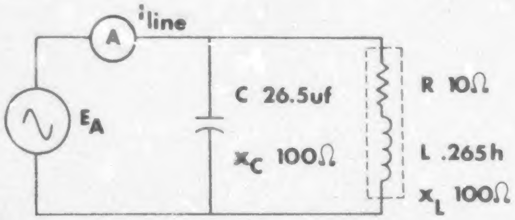
f_o = ______

- -

Answer: 159 KHz

Analyzing a Practical Tank Circuit

Consider the circuit shown below.



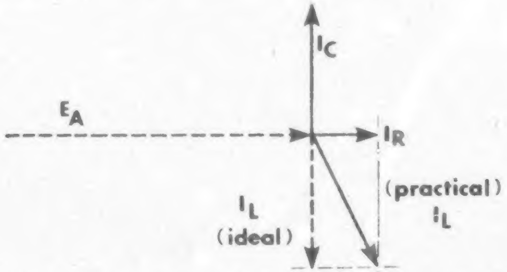
In practical applications, ideal circuit conditions can never be achieved. A practical circuit always has some resistance. This resistance is primarily due to the coil's windings and is depicted as a resistor in series with the coil in the above circuit.

Remember, that with X_L and R in series, the phase angle is between 0 and 90°.

This means that the current in the capacitor, and the current in the coil are not exactly 180° out of phase, and therefore, are not exactly equal and opposite.

Since the two currents do not completely cancel, there is some line current flowing to and from the source. Obviously, if the ratio of X_L to R is large, there is little difference between the two branch currents, and the net line current is small.

Note the effect shown on the diagram below.



Using Ohm's Law, we see that although the impedance of the parallel resonant circuit is not infinite as in the ideal tank, the total impedance is still very large.

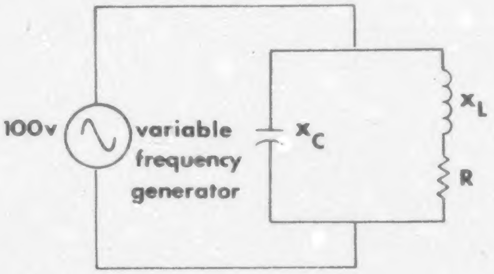
Note that this is exactly opposite to series resonance, since in series resonance circuits, we found the circuit current to be very high, and the impedance quite small.

Off-Resonance Effects in Practical Parallel Circuits

We have seen that when any LC circuit operates at its resonant frequency, the reactances are equal. We have seen that $X_L = X_C$ at only one frequency for a given L and C combination. The source sees a resistive load, whether R, L, and C are in series or in parallel.

At frequencies other than that of resonance, the effects of the reactances are not cancelled (X_L does not equal X_C), leaving one or the other dominant. In analyzing the off-resonance effects of the series RLC combination, we learned that when X_L is the larger, the circuit is inductive; when X_C is the larger, the circuit is capacitive.

Examine the parallel circuit shown below.



In our example, 60 Hz is the resonant frequency. Suppose we now change the variable-frequency generator to produce an output of 30 Hz. Since f has decreased, X_L decreases but X_C increases. At this lower frequency, the current through the coil increases while the current through the capacitor decreases. The total vector sum current supplied by the source is now largely inductive current. The circuit current lags the applied voltage (ELI). Hence, the circuit now appears inductive.

In general, we see:

> If the source frequency is below f_o, the parallel LC circuit appears inductive.

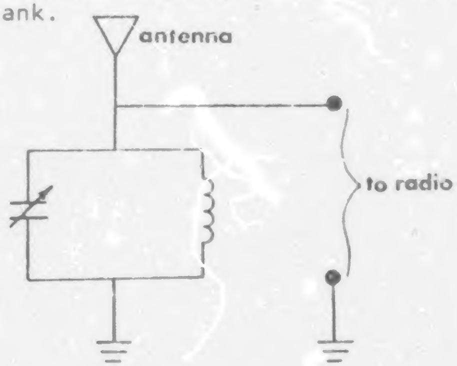
Now we raise the source frequency above f_o. X_L increases and X_C decreases. At this frequency, the current through the coil decreases, while the current through the capacitor increases. The current from the source is capacitive current and it leads the applied voltage (ICE). Therefore, the circuit behaves like a capacitive circuit.

In general, we see:

> If the applied frequency is above f_o, the parallel circuit appears capacitive.

Filters Using Parallel Resonant Circuits

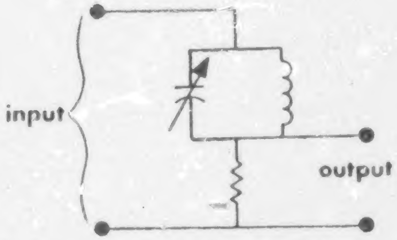
A parallel resonant LC circuit may be used as a very efficient bandpass filter. A very common use for a circuit like this is the tuning section of a radio receiver. The high impedance of the tank circuit at or near resonance develops a relatively high voltage to pass on to the following circuits. Here is a schematic of this type tank.



Frequencies higher than the resonant frequency are bypassed through the capacitor while those below resonance are passed through the inductor without producing a significant voltage drop.

By using a variable capacitor in the circuit, a tunable filter results, and the circuit can be adjusted to pick up one particular radio station while rejecting all others.

Should you desire to eliminate one frequency from a circuit (for example, a ham station creating interference with your TV), you can wire in a tank circuit like this one, and adjust it to reject the undesired signal.



This is called a band elimination filter, for it cuts out a limited range of frequencies without greatly affecting signals above or below its cutoff frequencies.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION

LESSON IV

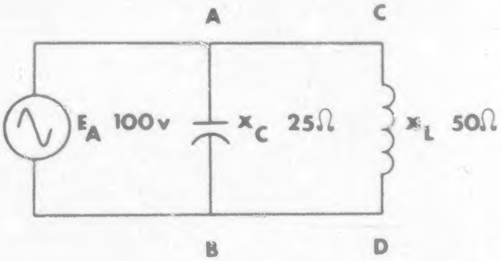
Resonant Frequency in Parallel AC Circuits

THIS PROGRAMMED SEQUENCE HAS NO TEST FRAMES.

The Ideal LC Circuit

So far in this module, we have discussed parallel RL, RC and RLC circuits and developed rules for determining the circuit values. By performing a variational analysis on these circuits, we have been able to observe characteristics of each type of circuit when circuit values are independently varied. A fourth type of parallel circuit containing only reactive elements displays interesting and unique characteristics. The parallel LC circuit cannot be totally without resistance for all circuits have resistance in their elements and wire. However, we will first discuss the ideal circuit with only reactive elements and later consider effects of resistance.

1. The basic ideal LC circuit is shown in the diagram below. Since it is a parallel circuit, ______________ is common and the voltage drop across AB __________ the drop across CD.



(voltage; equals)

2. The current flowing through each branch can be calculated from the equations:

$$I_C = \frac{E_a}{X_C}$$

$$I_L = \frac{E_a}{X_L}$$

and $$I_R = \frac{E_a}{R}$$

Using the values of voltage and impedance in the example, find:

I_C = ____________

I_L = ____________

I_R = ____________

* * *

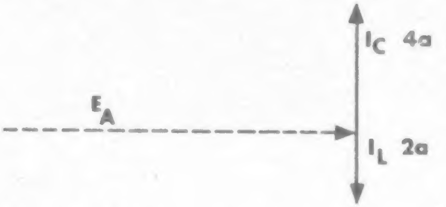
(4 a; 2 a; 0 a)

3. Remember that current flowing through resistance is in phase with voltage, inductive current ________ the applied voltage by 90°, and capacitive current ________ the applied voltage by 90°.

* * *

(lags: leads)

4. The following current vector diagram can be drawn. Current through the resistive elements is ________ in the ideal LC circuit example.



* * *

(zero)

5. As we did in parallel RLC circuits, we can add I_C and I_L when expressed in rectangular form. This is the same as saying that the capacitive and inductive currents are ________ degrees out of phase and can be added by finding their difference.

(180)

6. We must first write the three types of current in rectangular notation for the example circuit.

I_R = ________

I_L = ________

I_C = ________

(0 a + j0 a; 0 a - j2 a; 0 a + j4 a)

7. The total circuit current can be calculated by adding.

I_R = 0 a + j0 a

I_L = 0 a - j2 a

I_C = 0 a + j4 a

I_T = ________

(0 a + j2 a)

8. In this example, total current expressed in rectangular notation can be easily converted to polar form. Therefore, I_T = 0 a + j2 a in polar form becomes:

I_T = ________

(2 a $\underline{/90^\circ}$)

9. The total impedance can be calculated from Ohm's Law:

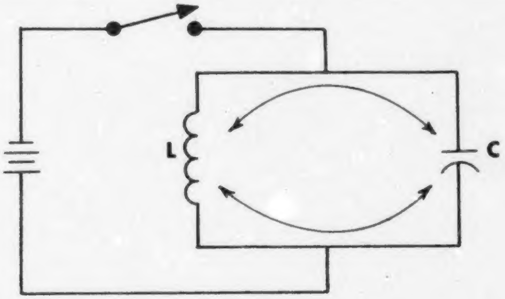
$$Z_T = \frac{E_a}{I_T}$$

$$Z_T = \_\_\_\_\_\_\_\_$$

- -

(50 Ω $\underline{/-90°}$)

10.

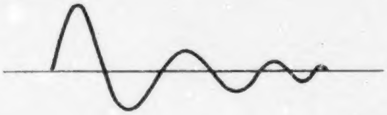


When the source of power is removed, current will continue to circulate between L and C. This effect is due to ________.

- -

(The discharging of the capacitor and the collapsing field of the inductor.)

11.



DAMPED WAVE

After the source of power is removed, the current will continue to circulate back and forth between the inductor and capacitor at a diminishing rate. This action is known as flywheel action or flywheel effect. The damped wave is caused by the ________ present in any practical circuit.

- -

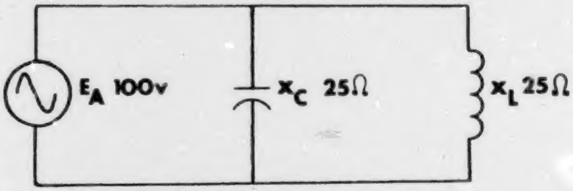
(Resistance)

Ideal LC Circuit at Natural Resonant Frequency

As we will see in the following frames, the ideal LC parallel circuit displays unique properties when the inductive and capacitive reactances are equal. The frequency of the applied voltage at which X_L equals X_C is defined as the natural resonant frequency of the circuit.

---

12. In the circuit illustrated below, X_L is equal to X_C.



Recall the equations for reactances:

$$X_C = \frac{0.159}{fC}$$

$$X_L = 2\pi fL$$

The frequency at which $X_C = X_L$ is defined as the ______________ ____________ frequency of the circuit, and is denoted as f_o.

- -

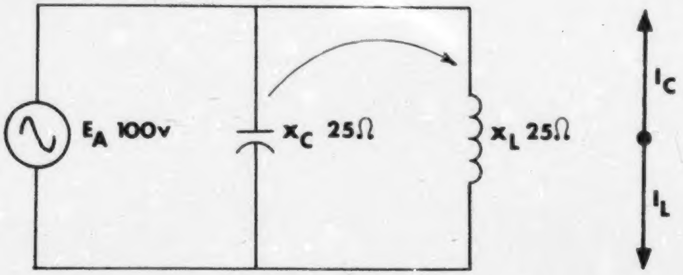
(natural resonant)

13. If we change the frequency from resonance, say increase it, X_C __________ and X_L _______________. In any case, X_C no longer equals X_L. A similar non-resonance phenomenon occurs if we decrease the applied frequency from f_o.

- -

(decreases; increases)

14. In the current vector diagram below, the inductive branch current is ____________ to the capacitive branch current.



(equal)

15. In the parallel LC circuit at the resonant frequency, I_C and I_L are equal and vectorially plotted in ____________________ directions.

(opposite)

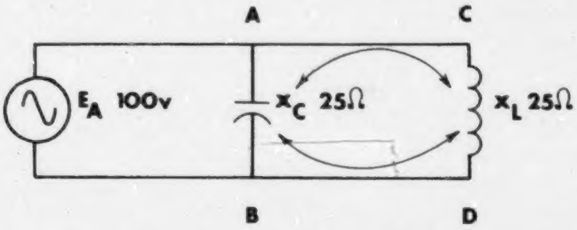
16. The total current calculated by vectorially adding I_C and I_L is ________________.

(zero)

17. When a parallel LC circuit is operating at resonance, the current flows from the capacitive branch to the inductive branch at one instant, while at another instant current flows from the inductive branch to the capacitive branch in the ______________ direction.

(opposite)

18. This alternating current appears as a loop of current in the tank circuit defined by ABCD in the diagram.



Even though current flows in the tank circuit, the total current flowing from the source is _______________.

- -

(zero)

19. At resonance $I_T = 0$, so the total impedance is infinite. Solve the circuit in frame 18 for Z_T.

Z_T = _______________

- -

$$\frac{E_a}{I_T} = \frac{100\ v}{0\ a} = \infty$$

(Note: Any number divided by zero is actually an undefined number. However, as current gets smaller and smaller, impedance must be increasing, so that when current is zero, impedance must have reached infinity.)

20. In review, the natural resonant frequency of an ideal parallel LC circuit occurs when capacitive reactance __________________ inductive reactance.

- -

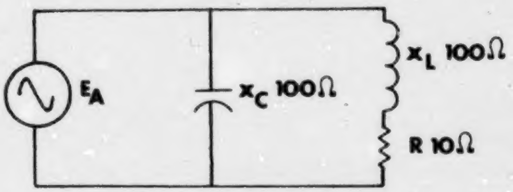
(equals)

21. In practical electrical circuits, resistance is always present. This resistance changes the special characteristics of resonance. The extent of the changes depends on the ratio of the inductive reactances to the resistance. Recall that low-Q coils have a relatively large ______________ resistance.

- -

(effective)

22. A practical LC circuit is shown in the diagram.

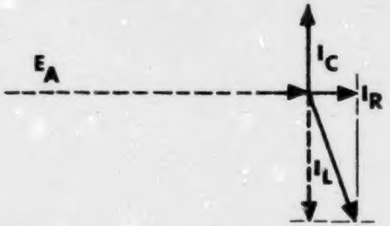


Recall that, with resistance in series with L, the current flowing through the inductive branch is no longer 90° out of phase, but lags voltage by a phase angle between ___________ and 90°.

- -

(0°)

23. We represent this change in the phase angle inductive current, as indicated on the vector diagram below.



Right away, we expect the total current at resonance to be __________, due to the presence of I_R.

- -

(greater than zero)

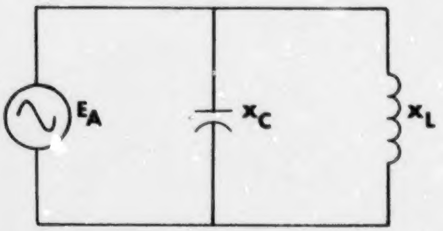
24. In the ideal parallel LC circuit at resonance, I_T is zero, and the total impedance is infinite. In a practical circuit, if R is small, the total current (or I_{LINE}) is also small, and Z_T is not infinite but remains ________________.

(large)

25. Recall that resonance occurs when the inductive reactance, X_L, is ________________ to the capacitive reactance, X_C.

(equal)

26. In the circuit example, assume that we have fixed values of L and C. The reactive impedances are defined by the equations:



$$X_L = 2\pi f_o L$$

$$X_C = \frac{1}{2\pi f_o C}$$

In the resonant condition, we can equate the two, $X_L = X_C$, and substitute:

$$2\pi f_o L = \frac{1}{2\pi f_o C}$$

The resonant frequency can be calculated from this equation, using $2\pi = 6.28$.

$f_o =$ __________

$$\frac{0.159}{\sqrt{LC}}$$

27. If we now increase the applied frequency above f_o, keeping all other factors constant, X_L __________ and X_C __________.

- -

(increases; decreases)

28. Since X_L is greater than X_C, the current flowing through X_L is ______________ than the capacitive current. Look at Ohm's Law ($I_L = \frac{E_a}{X_L}$) for the solution.

- -

(smaller)

29. We can say, therefore, that when the applied frequency is above f_o, I_C is larger than I_L and the circuit appears ______________.

- -

(capacitive)

30. If we decrease the applied frequency below f_o, X_L __________ and X_C __________.

- -

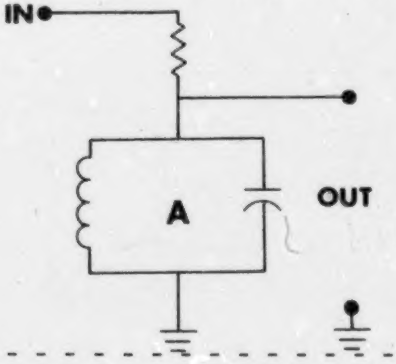
(decreases; increases)

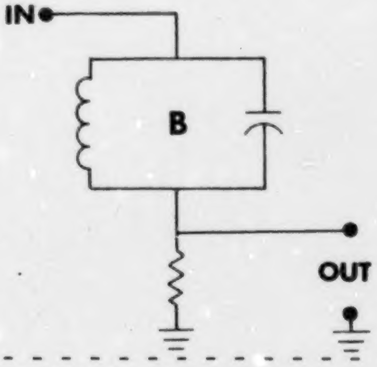
31. From Ohm's Law, $I_C = \frac{E_a}{X_C}$ and $I_L = \frac{E_a}{X_L}$. I_L is ______________ than I_C when the parallel LC circuit operates below resonance. In this case, the circuit appears ______________.

- -

(larger; inductive)

32. Parallel resonant circuits are commonly used as filters to select or reject narrow bands of frequencies. Based on what you know about filters and parallel resonant circuits, which of these is a bandpass filter?

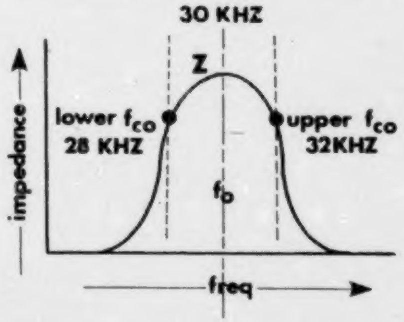




(A)

33. Choice A is correct, for the parallel LC circuit has a high impedance at or near resonance only. At any frequency lower than its lower f_{co}, the signal is shorted through the inductor to ground without developing a usable output voltage.

A signal at a frequency of 20,000 Hz ________ useful at the output of a bandpass filter with the resonance curve shown?
is/is not



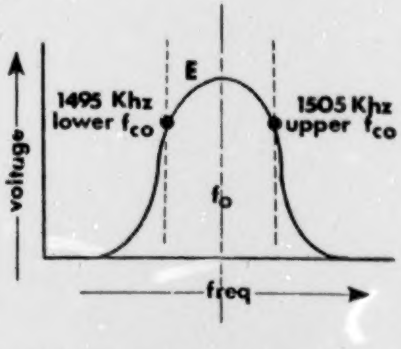
(is not)

34. Frequencies higher than the upper f_{co} are bypassed through the capacitor and again no usable output is developed. Using the graph from frame 33, determine which component offers low impedance to a frequency of 65 KHz?

(the capacitor)

35. Because the impedance at (or near) resonance is high, an output voltage develops across the tank circuit in this region, and a graph of voltage versus frequency looks much like the impedance curve.

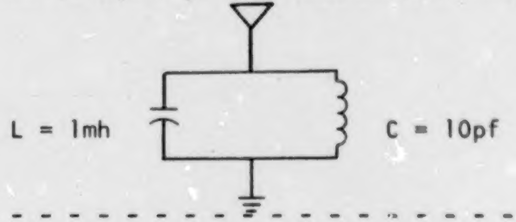
What is the resonant frequency of the circuit whose output is graphed here?



(1500 KHz)

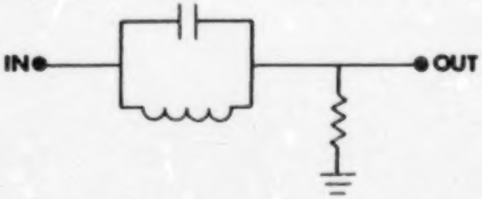
36. A very common use for bandpass filters is the tuning circuit used in a radio. Placing a variable capacitor in place of the fixed capacitor allows the resonant frequency to be varied so that one station will be received while all others are rejected.

What station (frequency) does this circuit select?



(1590 KHz)

37. A parallel LC circuit may also be used to eliminate an unwanted frequency, such as interference to the picture of your TV. This is done by wiring the circuit a little differently:



Here, the voltage developed across the resistor (IR) is ______________ at resonance because the impedance of the tank
maximum/minimum
circuit is ______________.
maximum/minimum

- -

(minimum; maximum)

38. The circuit shown in frame 37 is a band reject filter, for it eliminates ______________________ of frequencies from the
one band/all but one band
output.

- -

(one band)

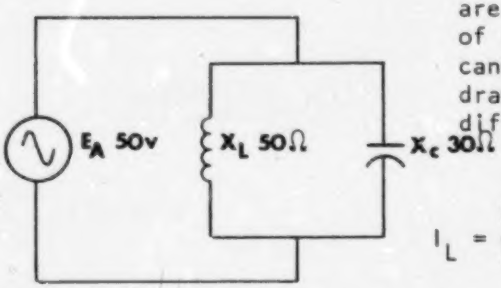
YOU MAY NOW TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

SUMMARY

LESSON IV

Resonant Frequency in Parallel AC Circuits

To further your understanding of how components react to applied frequency when connected in parallel, we will now idealize (disregard all circuit resistance) a circuit containing a capacitor and a coil.

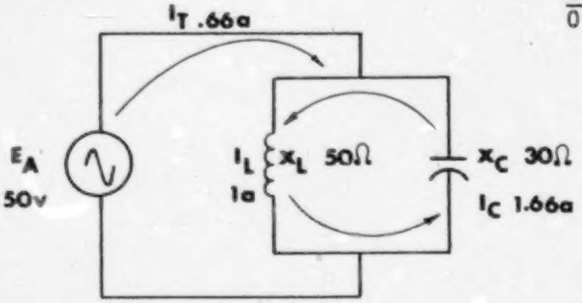


Consider first that I_L and I_C are 180° out of phase. Because of this phase difference, they cancel each other and the current drawn from the source equals the difference between the two ($I_C - I_L$).

$$I_L = \frac{50 \angle 0^\circ}{50 \angle +90^\circ} = 1 \text{ a } \angle -90^\circ$$

$$I_C = \frac{50 \angle 0^\circ}{30 \angle -90^\circ} = 1.66 \text{ a } \angle +90^\circ$$

$$I_T = \begin{array}{r} 0 - j1 \\ \underline{0 + j1.66} \\ 0 + j0.66 \end{array}$$



As you can see from the illustration, 1 amp of current passes back and forth between the coil and the capacitor with the source supplying the extra 0.66 amp to maintain I_C at its Ohm's Law value.

Because of the transfer of energy between the two reactive devices, total current is less than either I_C or I_L.

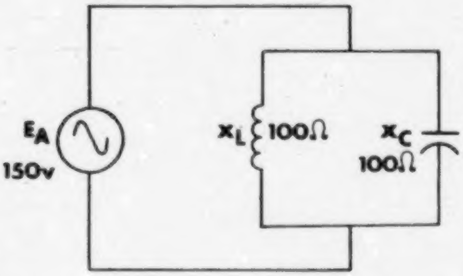
By applying Ohm's Law, we can solve for Z_T.

$$Z_T = \frac{E_a}{I_T} = \frac{50}{0.66} = 75.7\ \Omega$$

Note that Z_T is greater than either X_C or X_L. This is an important aspect of a parallel LC circuit and we will discuss it further.

When the source of power is removed, current will continue to circulate between the inductor and the capacitor forming a damped wave. This action is known as flywheel action or flywheel effect.

Circuits which contain inductance in parallel with capacitance are called tank circuits. The current which flows within the tank (back and forth between the capacitor and the coil) is known as circulating current (I_{CIR}) or tank current (I_{TANK}). The current which the source supplies is called line current (I_{LINE}). Line current is merely another name for I_T.



By looking at the accompanying schematic and using a little imagination, you should be able to determine I_{LINE} and circuit impedance. You can see that $X_L = X_C$. This means that $I_L = I_C$.

Due to the 180° phase shift between the equal branch currents, there is a complete canceling effect and I_{LINE} is zero.

$$I_L = \frac{150}{100 \underline{/+90^\circ}} = 1.5 \text{ a } \underline{/-90^\circ}$$

$$I_C = \frac{150}{100 \underline{/-90^\circ}} = 1.5 \text{ a } \underline{/+90^\circ}$$

$$I_{LINE} = \begin{array}{r} 0 - j1.5 \\ \underline{0 + j1.5} \\ 0 + j0 \end{array}$$

With line current known to be zero, circuit impedance can be computed by Ohm's Law.

$$Z = \frac{E}{I} = \frac{150}{0} = \text{infinity } (\infty)$$

(Remember, this is an ideal situation.)

When these conditions exist:

$$X_C = X_L$$

$$I_C = I_L$$

$$I_{LINE} = 0$$

$$Z =$$

the circuit is said to be at its natural resonant frequency.

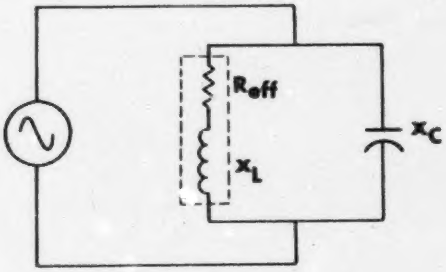
Natural resonance always occurs when the frequency of the applied voltage is such as to make $X_L = X_C$. This is true for both series and parallel circuits.

Recall that the formula used to find resonance in series circuits is

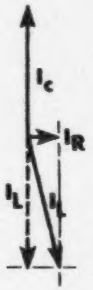
$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{ or } \frac{0.159}{\sqrt{LC}}$$

The same formula is used to find the resonant frequency of a parallel network because we are looking for the same circuit condition, $X_L = X_C$.

In practical applications, ideal circuit conditions can never be achieved. The resistance present in the coil's windings upsets the balance between current through the inductive and capacitive branches.



The two currents are not exactly equal and opposite; therefore, there is some line current drawn from the source.



The dotted line (I_L) represents what I_L would be under ideal conditions; the solid vector (I_L) represents the actual current through the inductive branch. Note that I_L is slightly less than I_C and the two currents are not exactly 180° out of phase.

Due to the resistance in the circuit, I_{LINE} increases from the ideal value of 0, and impedance decreases from infinity to some finite value.

A more practical statement of conditions at resonance of a parallel circuit is:

$$X_L = X_C$$

$$I_{LINE} = \text{minimum value}$$

$$Z_T = \text{maximum value}$$

At the resonant frequency, a parallel LC circuit appears purely resistive to the source $(\underline{/\theta^\circ} = 0^\circ)$ because the only current drawn is due to the circuit resistance. If frequency is varied from resonance, the reactances (X_L and X_C) are no longer equal.

When the applied frequency drops below f_o, inductive reactance decreases and inductive current increases.

At the same time, capacitive reactance increases and capacitive current decreases.

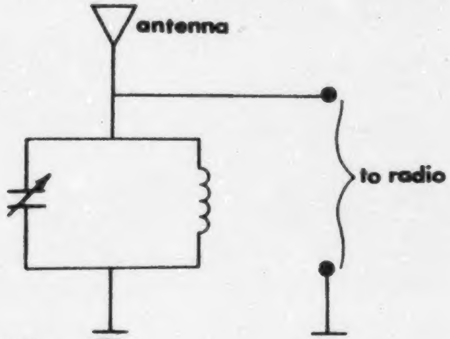
Since the inductive current is the greater, the circuit appears inductive at frequencies below resonance.

When the applied frequency varies above the resonant frequency, inductive reactance increases and inductive current decreases. Capacitive reactance decreases and capacitive current increases.

Since capacitive current is the greater, the circuit appears capacitive at frequencies above resonance.

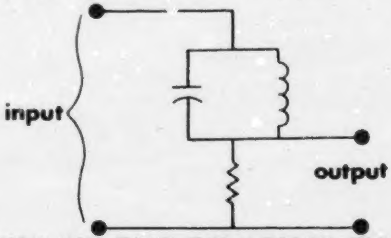
Remember, at resonance I_{LINE} is minimum and Z is maximum, so I_{LINE} will increase whether frequency goes above or below resonance.

One use for this type of circuit is a band-pass filter. This filter develops a significant voltage only if the applied frequency is near its resonant frequency. In this way, we get a circuit which selects a very narrow band of frequencies and rejects all others. A variable capacitor is used to build a widely-used tuned circuit for radio receivers:



The capacitance is varied to select the desired station while all other frequencies fail to develop usable outputs.

Wired like this, the parallel LC network forms a band-reject filter:



AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, SELECT ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-14a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON V

Effective Resistance in RL Parallel Circuits

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON V

Effective Resistance in RL Parallel Circuits

In this lesson you will study and learn about the following:

- practical RL circuits
- solving for current
- effect of Q of a coil in parallel RL circuits

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON V

Effective Resistance in RL Parallel Circuits

To learn the material in this lesson, you have the option of choosing, according to your experience and preferences, any or all of the following:

STUDY BOOKLET:

- Lesson Narrative
- Programmed Instruction
- Lesson Summary

ENRICHMENT MATERIAL:

NAVPERS 93400A-1b. "Basic Electricity, Alternating Current." Fundamental of Electronics. Bureau of Naval Personnel. Washington, D.C.: U.S. Government Printing Office, 1965.

YOU MAY NOW STUDY ANY OR ALL OF THE RESOURCES LISTED ABOVE. YOU MAY TAKE THE PROGRESS CHECK AT ANY TIME.

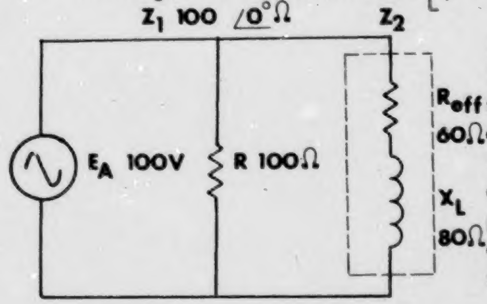
NARRATIVE

LESSON V

Effective Resistance in RL Parallel Circuits

Practical RL Circuits

Up to this point in parallel circuit analysis, we have considered the coil as ideal. In other words, we have assumed that the coil was purely inductive, but in reality we knew this to be untrue. A coil also has resistance -- the effective resistance of the coil. When the effective resistance of a coil is high in relation to X_L, it cannot be ignored.



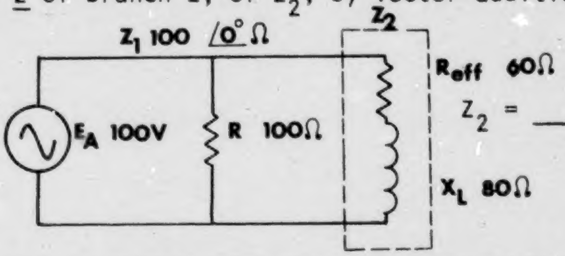
If the Q of a coil ($Q = \frac{X_L}{R}$) is less than 10, the effective resistance must be taken into consideration. The branch of the circuit which contains the coil is no longer considered purely inductive because, in effect, there is resistance in series with the coil.

When a leg of the parallel network has both R and X_L, then I through the leg no longer lags E by 90° as it does in a purely inductive leg. The phase angle changes, and as with any series RL circuit, the ohmic values of R and X_L cannot be arithmetically added to find the total impedance in the branch.

Before we can determine the current flow through this resistive-reactive branch of the network, it is necessary to find total impedance in the branch.

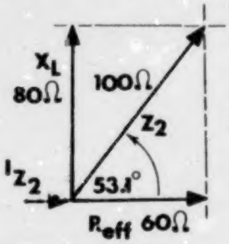
Remember that we can vector to find Z_T in series AC circuits only; however, the coil and the effective resistance are in series in this branch of the parallel circuit.

Find Z of branch 2, or Z_2, by vector addition.



Z_2 = ____________

Impedance vector diagram:



As this is a 3-4-5 triangle, you know Z2 = 100 ohms, and $\underline{/\theta}$ = 53.1°.

Solving for Current

Now we know that the impedance in branch 2 is 100 ohms $\underline{/53.1°}$. We also know that because the coil and the effective resistance are in series, the same current flows through each one. Current through Z2 is determined as follows:

$$I_{Z2} = \frac{E_a}{Z2}$$

$$I_{Z2} = \frac{100\ \underline{/0°}}{100\ \Omega\ \underline{/53.1°}}$$

(Remember E_a is the common reference in a parallel network.)

$$I_{Z2} = 1\text{ a } \underline{/-53.1°}$$

(Note: Here we are saying that the current in the inductive leg is lagging E_a by 53.1°. This makes sense because in a purely inductive circuit $\underline{I}$ lags $\underline{E}$ by 90°, but because we have resistance in series with the coil, the phase angle has decreased from 90° to 53.1°.)

Current through Z1 is determined in this manner:

$$I_{Z1} = \frac{E_a}{R}$$

$$I_{Z1} = \frac{100\text{ v } \underline{/0°}}{100\ \Omega\ \underline{/0°}}$$

$$I_{Z1} = 1\text{ a } \underline{/0°}$$

Adding the Branch Currents

To add the individual branch currents to obtain total current, we must express them in rectangular notation.

$$I_{Z2} = 1 \text{ a } \underline{/-53.1^\circ} \quad \text{(polar)}$$

$$I_{Z2} = .6 - j0.8 \text{ a} \quad \text{(rectangular)}$$

Adding I_{Z1} and I_{Z2} to find I_T:

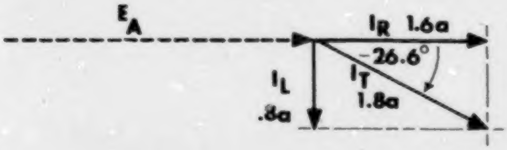
$$I_{Z1} = 1.00 + j0 \quad \text{a}$$

$$\underline{I_{Z2} = 0.6 \; - j0.8 \text{ a}}$$

Therefore: $I_T = 1.6 \; - j0.8$ a

Now we have I_T expressed in rectangular form.

Constructing Current Vector Diagram



Since we know the values of resistive and inductive current, we can use the formula

$$\text{TAN } \underline{/\theta} = \frac{I_L}{I_R} = \frac{0.8}{1.6} = .5000.$$

In the trig tables, we find: $\underline{/\theta} = 26.6^\circ$.

$$\text{COS } \underline{/\theta} = .8942$$

Then $\text{COS } \underline{/\theta} = \frac{I_R}{I_T}$; transposed, $I_T = \frac{I_R}{\text{COS } \underline{/\theta}}$ or $I_T = \frac{1.6}{.8942} = 1.8$ a.

In polar form, I_T is 1.8 a $\underline{/-26.6^\circ}$

Relationship of Q in Parallel Circuits

In many applications of parallel networks, keeping power consumption at a minimum is important. The greater the P_t of a given circuit, the more power consumed.

When effective resistance of a coil in a parallel RL circuit increases, then the power consumed increases. When the circuit consumes more power, the inductor stores less power. A low-Q coil dissipates more power than one of equal inductance with a high-Q, because the effective resistance is greater in the low-Q coil.

As long as the Q of a coil remains 10 or above, we need not concern ourselves with the phase shift of the inductive leg because the effective resistance is negligible. However, if the Q of the coil is less than 10, phase shift must be considered.

For your purposes in this school, you will disregard the effective resistance of the coil unless otherwise directed.

AT THIS POINT, YOU MAY TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY METHOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

PROGRAMMED INSTRUCTION

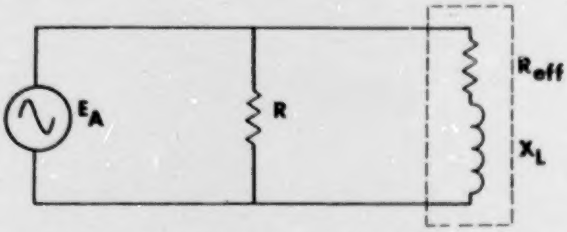
LESSON V

Effective Resistance in RL Parallel Circuits

THIS PROGRAMMED SEQUENCE HAS NO TEST FRAMES.

An inductor possesses both resistance and impedance. To simplify calculations, we can represent the effective resistance of a coil as a pure resistance in series with the coil. With the resistance "removed", the impedance of the coil is again purely reactive and its value is represented by X_L.

1. In the parallel RL circuit shown, four major elements are indicated although the actual circuit has only three parts -- voltage source, resistor, and ________.



- -

(inductor or coil)

2. R and X_L are expressed in ohms. The effective resistance of the coil is expressed in ________.

- -

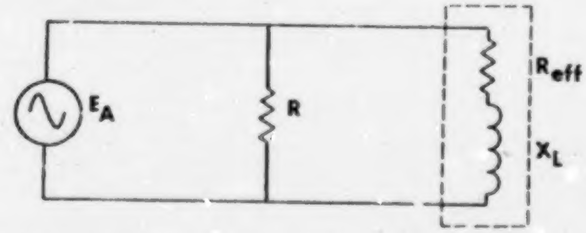
(ohms)

3. To calculate voltage and current values for the parallel circuit illustrated, we can start with the procedure developed in the last lesson for circuits with purely resistive and inductive branches. For the resistive branch, we calculate the branch current according to the equation I_R = ________, since voltage is common and the voltage drop across the resistor is equal to ________.

- -

$\frac{E_a}{R}$, E_a

4.

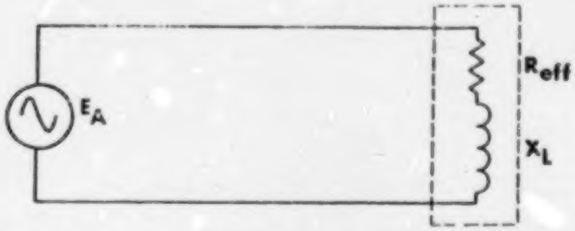


Similarly, the voltage drop across the reactive branch is equal to the __________ voltage.

- -

(source)

5. Neglecting the resistive branch for a moment, we can solve for current flowing through the inductive branch as if it were a series circuit as shown. With R_{eff} and X_L, this is simply a __________ RL circuit like you studied in Module Twelve.



- -

(series)

6. Since R_{eff} is shown to be in series with the coil, the impedance of the branch (Z2) must be equal to the __________ sum of R_{eff} and X_L.

- -

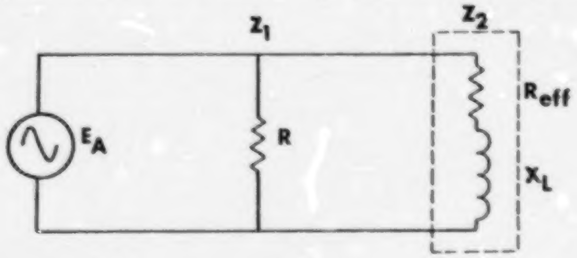
(vectored)

7. You know the voltage drop across the resistive element is in phase with the current. The voltage drop across the inductive element is out of phase and ____________ current by 90°.

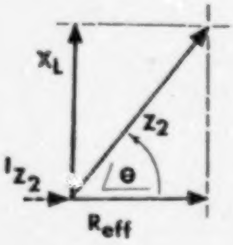
(leads)

8. Recall that in a series RL circuit we cannot directly add impedances to obtain the total impedance of the circuit. Z_T must be obtained by vector addition of ________ and __________.

Draw the impedance vector diagram for the inductive branch in the circuit illustrated and label the impedance and phase angle.



(R_{eff}; X_L)



9. Calculate the impedance and phase angle of the inductive branch when R_{eff} and X_L are both 10 ohms.

Z_T = ________

$\angle\theta$ = ________

(14 Ω; 45°)

10. The impedance of the inductive branch can be represented in rectangular notation as:

_________ ohms.

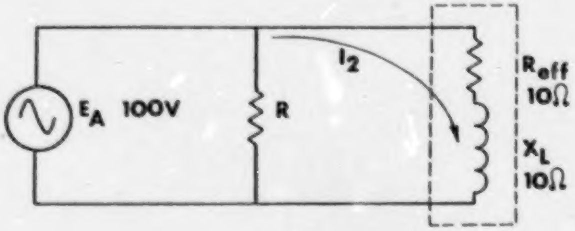
- -

(10 + j10)

11. The same current, I_2, flows through X_L and R_{eff}. For a source voltage of 100 volts, we can solve for I_2 using Ohm's Law,

$I_2 = \frac{E_a}{Z_2}$ (impedance of the inductive branch).

When we divide 100 volts by 14 ohms to obtain I_2, we must consider the _____________ difference between voltage and current.



- -

(phase)

12. Remember that we are solving this branch for impedance and current in order to ultimately solve the parallel RL circuit. For the parallel circuit, E_a is the common reference and therefore has a ____________ phase angle. E_a written in polar form is _____________.

- -

(zero; 100 v $\underline{/0^\circ}$)

13. With $E_a = 100 \text{ v } \underline{/0^\circ}$ and $Z_T = 14\ \Omega\ \underline{/45^\circ}$, the current I_2 flowing through the inductive branch is ____________.

- -

($7.1 \text{ a } \underline{/-45^\circ}$)

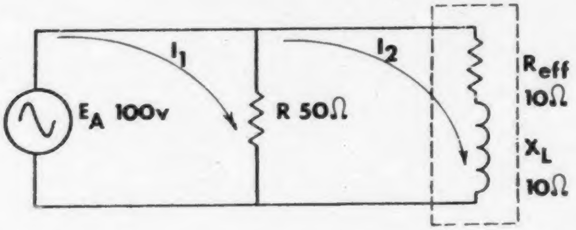
14. Recall that in a purely inductive circuit, voltage leads current by 90°. This is the same as current lagging voltage by 90°.

For the inductive branch of the example circuit, we have found that current lags the voltage by 45°. This decrease in the phase angle from 90° is caused by the ______________ in the circuit.

- -

(resistance of the coil or R_{eff})

15. Current flowing through the resistive branch can be calculated directly from Ohm's Law.



I_1 = __________

- -

$$\frac{100 \text{ v } \underline{/0^\circ}}{50\ \Omega\ \underline{/0^\circ}} = 2 \text{ a } \underline{/0^\circ}$$

16. For the parallel RL circuit, we have calculated the branch current to be:

$$I_1 = 2 \text{ a } \underline{/0^\circ}$$

$$I_2 = 7.1 \text{ a } \underline{/-45^\circ}$$

To find the total current for the circuit, I_T, we combine I_1 and I_2 by ____________ addition since the two currents are out of phase.

- -

(vector)

17. Since the branch currents are not 90° out of phase with each other ($I_1 = 2\ \underline{/0^\circ}$, $I_2 = 7.1\ \underline{/-45^\circ}$) combining them is a bit more difficult. To combine these to currents they must first be expressed in ____________ notation.

- -

(rectangular)

18. I_1 written in rectangular notation is: ____________

- -

(2 + j0)

19. Branch current I_2, consists of a resistive component (I_R) and a reactive component (I_L). To change I_2 from it's polar form (7.1 $\underline{/45^\circ}$) to rectangular form, follow the procedure learned in Module Twelve, Lesson III.

resistive element (I_R) = (Cosine $\underline{/\theta}$) (Hyp)

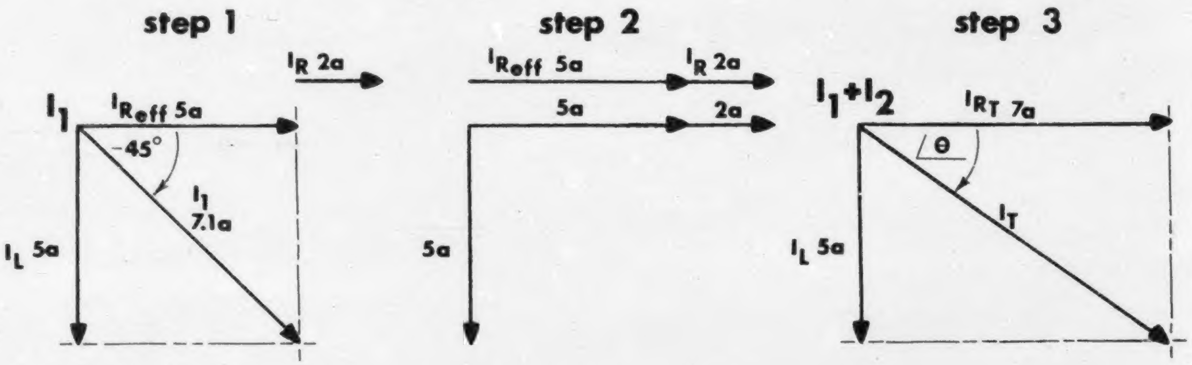
reactive element (I_L) = (SIN $\underline{/\theta}$) (Hyp)

Convert I_2 to rectangular notation ____________________.

- -

(5 - j5)

20. The procedure for adding the branch current vectors can be visualized from these diagrams.


step 1
step 2
step 3
I_R 2a
I_1
I_{Reff} 5a
-45°
I_1
7.1a
I_L 5a
I_{Roff} 5a
I_R 2a
5a
2a
5a
I_1+I_2
I_{RT} 7a
/θ
I_T
I_L 5a

1. Convert I_1 and I_2 from polar notation to rectangular notation.

2. Add I_1 and I_2 to determine the total current expressed in rectangular notation.

3. Convert I_T from rectangular notation to polar notation.

Solve the example problem for total current and express in polar notation. I_T = ______________.

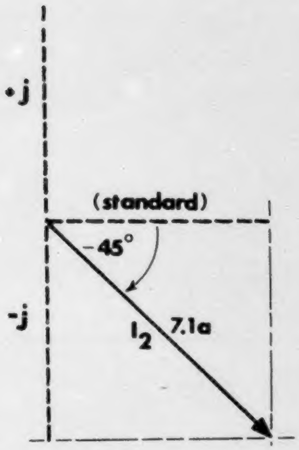
(8.6 a /-35.5°)

IF YOU DID NOT GET THE CORRECT ANSWER, GO TO FRAME 21. IF YOU WERE CORRECT, GO TO FRAME 29.

21. From the diagram, branch current, I_1, is in phase with the common voltage and possesses a zero phase angle. The polar representation, I_1 = 2 a /0°, can be directly written in rectangular form: I_1 = __________________.

(2 a + j0)

22. Converting I_2 to rectangular notation is slightly more complicated and can best be visualized by looking at the drawing showing one component in the standard direction and one component in the _____________ direction.



$I_2 = 7.1$ a $\underline{/-45^\circ}$

- -

(-j)

23. We can calculate the component values of the inductive branch current $(I_R - jI_L)$ by applying the trigonometric functions to the vector diagram shown in Frame 20.

The resultant is represented by the current I_2 (7.1 a $\underline{/-45^\circ}$).

To find the resistive value (I_R), apply the cosine function

$$\text{Cosine} = \frac{I_R}{I_2}$$

by rearranging the equation we get

$$I_R = \text{Cosine } I_2$$

Cosine $\underline{/-45^\circ} = 0.707$

$$I_2 = 7.1 \text{ a}$$

Therefore, the resistive element of I_2 = ___________.

- -

(COS $\underline{/-45^\circ}$ × 7.1 a or 5 a; Note: We are not attempting to say that you vector current in a series circuit. We are simply converting a vector representing that current from polar notation to rectangular notation.)

24. Similarly, the magnitude of I_2 in the -j direction can be calculated from the trig relationship:

$$\text{SIN } \underline{/\theta} = \frac{\text{inductive}}{I_2}$$

Then, the inductive element of I_2 = ______________

- -

(SIN $\underline{/-45°}$ x 7.1 a or - j5 a)

25. Now we can write I_2 in the rectangular notation.

I_2 = ________________

- -

(5 a - j5 a)

26. The total current for the circuit in rectangular notation is the sum of I_1 and I_2.

I_1 = 2 a + j0

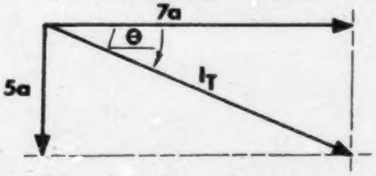
I_2 = $\underline{\text{5 a - j5 a}}$

I_T = __________

- -

(7 a - j5 a)

27. Now that we have a value for I_T in rectangular notation, we can convert to polar form using trigonometric functions.



The $\underline{/\theta}$ can be calculated using the tangent.

TAN $\underline{/\theta}$ = __________

From the trig tables:

$\underline{/\theta}$ = ______________

- -

$\frac{5\text{ a}}{7\text{ a}}$ = 0.7143; -35.5°

28. The magnitude of I_T can be found using the equation for the cosine of the phase angle:

I_T = ________________

- -

(8.6 a $\underline{/-35.5°}$)

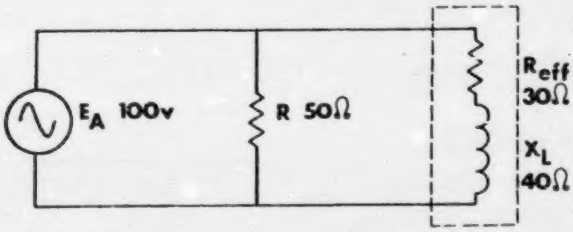
29. Recall that any coil contains some resistance. A high-Q coil contains __________ resistance or R_{eff} than a low-Q coil.

A rule of thumb to follow is that if the Q of a coil is greater than 10, the effective resistance is negligible and can be considered ____________________.

- -

(lower; zero)

30. To prove the statement made in Frame 29. We will work a problem first with a low Q coil, then in the following frame replace it with a high-Q coil and rework the circuit.



1. Solve for Q _______________
2. Z of branch 2 _____________
3. I_2 (polar) ______________
4. I_2 (rectangular) __________
5. I_1 ________________________
6. I_T ________________________
7. Z_T ________________________

- -

(1. $Q = \frac{X_L}{R_{eff}} = \frac{40}{30} = 1.33$

2. $Z2 = 30 + j40 = 50\ \Omega\ \underline{/+53.1°}$

3. $I_2 = \frac{E_a}{Z2} = \frac{100\ \underline{/0°}\ v}{50\ \underline{/+53.1°}} = 2\ a\ \underline{/-53.1°}$

4. $I_R = (\cos \underline{/-53.1^\circ})(I_2)$ $\quad I_L = (\sin \underline{/-53.1^\circ})(2)$

$(0.6004)(2) \qquad (0.7997)(2)$

$1.2 - j1.6$

5. $I_1 = \frac{E_a}{Z1} = \frac{100 \underline{/0^\circ}}{50 \underline{/0^\circ}} = 2 \underline{/0^\circ}$

6. $I_1 = 2 + j0$

$I_2 = 1.2 - j1.6$

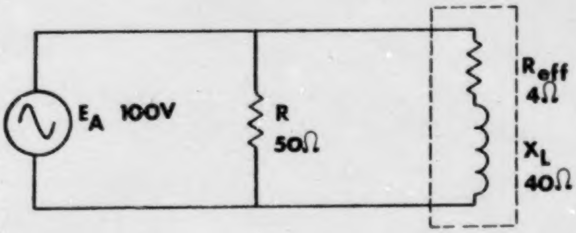
$I_T = 3.2 - j1.6$ (REC)

$3.58 \underline{/-26.6^\circ}$ a (polar)

7. $Z_T = \frac{E_a}{I_T} = \frac{100 \underline{/0^\circ} \text{ v}}{3.58 \underline{/-26.6^\circ}} = 28\ \Omega \underline{/+26.6^\circ}$)

---

31. Replace the low-Q coil with a high-Q.



Solve for:

1. Q = ____________
2. Z2 = ____________
3. I_2 (polar) = ____________
4. I_2 (Rec) = ____________
5. I_1 = ____________
6. I_T = ____________
7. Z_T = ____________

- -

(1. $Q = \frac{X_L}{R_{eff}} = \frac{40}{4} = 10$

2. $Z2 = 4 + j40 = 40.28\ \Omega \underline{/+84.3^\circ}$

3. $I_2 = \frac{E_a}{Z2} = \frac{100 \underline{/0^\circ} \text{ v}}{40.28 \underline{/+84.3^\circ}} = 2.48 \text{ a } \underline{/-84.3^\circ}$

4. $I_R = (\cos \underline{/-84.3°})(2.48) = 0.236$ a

$I_L = (\sin \underline{/-84.3°})(2.48) = 2.37$ a

5. $I_1 = \frac{E_a}{R} = \frac{100 \underline{/0°}}{50 \underline{/0°}} = 2 \underline{/0°}$ a

6. $$\begin{array}{l} I_1 = 2 \quad + j0 \\ I_2 = .246 - j2.47 \\ \hline I_T = 2.246 - j2.47 \text{ (REC)} \\ \quad 3.34 \underline{/-47.7°} \text{ (Polar)} \end{array}$$

7. $Z_T = \frac{E_a}{I_T} = \frac{100 \underline{/0°}}{3.26 \underline{/-46.8°}} = 30.6 \underline{/+46.8°}$)

---

32. By examining the results computed for branch 2 in frame 28, (you can see the Z2 is very nearly equal to ______________ and $\underline{/\theta}$ is quite close to ________°. The effects of R_{eff} are negligible so the branch can be considered purely ____________.

- -

(X_L, 90°, inductive)

YOU MAY NOW TAKE THE PROGRESS CHECK, OR YOU MAY STUDY ANY OF THE OTHER RESOURCES LISTED. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANY MEHTOD OF INSTRUCTION YOU WISH UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

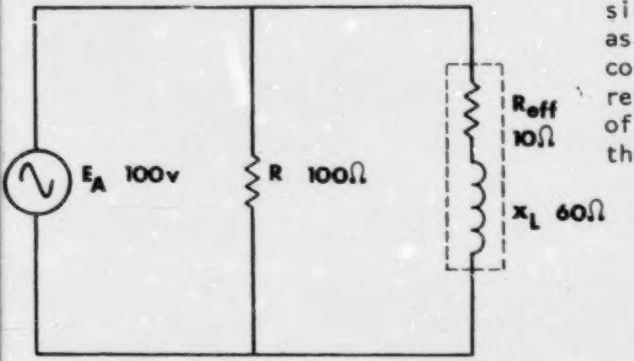
SUMMARY

LESSON V

Effective Resistance in RL Parallel Circuits

In Module Thirteen you were introduced to effective resistance and its effect on the operation of a coil. The R_{eff} of the coil acts as a resistance in series with a pure inductance and reduces the inductive phase angle from the ideal 90° to something less than this, depending on the value of R_{eff} in respect to X_L.

If the ratio of X_L to R_{eff} (Q) is less than 10, the effect of R_{eff} on circuit action must be taken into consideration. For example:



In this circuit, the effective resistance of the coil is represented as a resistor in series with the coil. The effect of this series resistance increases the impedance of the inductive branch and reduces the phase angle from the ideal 90°.

The impedance of the resistive branch is equal to the value of the resistor ($Z_1 = 100\Omega$). The impedance of the inductive branch is $Z_2 = 10 + j60$ or $61\Omega \underline{/80.6°}$.

The branch currents can now be computed by Ohm's Law:

$$I_1 = \frac{100v}{100\Omega} = 1 \text{ a } \underline{/0°} \qquad I_2 = \frac{100v}{61\Omega \underline{/80.6°}} = 1.64 \text{ a } \underline{/-80.6°}$$

Before the branch currents can be added, they must be converted from polar form to rectangular form.

Branch one current = $1 \underline{/0°} = 1 + j0$

Branch two current = $1.64 \underline{/-80.6°} = 0.268 - j1.61$

$$\text{Total current} = \begin{array}{r} 1. \quad + j0 \\ \underline{0.268 - j1.61} \\ 1.268 - j1.61 \end{array}$$

Total circuit current and circuit phase angle can now be found by converting the rectangular representation for I_T to polar form.

$$I_T = 2.15 \text{ a } \underline{/-51.8°}$$

Recall that Q is determined by the ratio of reactance to resistance within the coil; $Q = \frac{X_L}{R}$. A coil with a low X_L to R_{eff} ratio has a low Q and therefore cannot be regarded as a pure inductance.

As long as the Q of a coil remains 10 or above, the effective resistance is considered negligible and the shift in $\underline{/\theta}$ in the inductive branch need not be considered. If the Q of the coil is below 10, however, the effect of its resistance on the circuit is significant and must be considered.

AT THIS POINT, YOU MAY TAKE THE LESSON PROGRESS CHECK, OR YOU MAY STUDY THE LESSON NARRATIVE OR THE PROGRAMMED INSTRUCTION OR BOTH. IF YOU TAKE THE PROGRESS CHECK AND ANSWER ALL OF THE QUESTIONS CORRECTLY, GO TO THE NEXT LESSON. IF NOT, STUDY ANOTHER METHOD OF INSTRUCTION UNTIL YOU CAN ANSWER ALL THE QUESTIONS CORRECTLY.

NAVPERS 94558-14a

BASIC ELECTRICITY AND ELECTRONICS
INDIVIDUALIZED LEARNING SYSTEM

MODULE FOURTEEN

LESSON VI

Parallel Resonance Experiments

Study Booklet

Bureau of Naval Personnel
January 1972

OVERVIEW

LESSON VI

Parallel Resonance Experiments

In this lesson you will conduct some experiments with a parallel RCL circuit operating at a range of frequencies including the resonant frequency.

You will require:

- -NEAT Board 8
- -Audio Signal Generator
- -Multimeter

You will perform the following experiments:

- -determining f_o by I_{LINE}
- -determining f_o by I_C and I_L
- -effects of changing Q
- -effect of f_o of varying C

BEFORE YOU START THIS LESSON, PREVIEW THE LIST OF STUDY RESOURCES ON THE NEXT PAGE.

LIST OF STUDY RESOURCES

LESSON VI

Parallel Resonance Experiments

Since this lesson consists of experiments, there is only the narrative. There are no other study resources and no progress checks.

TURN THE PAGE AND BEGIN THE NARRATIVE.

NARRATIVE

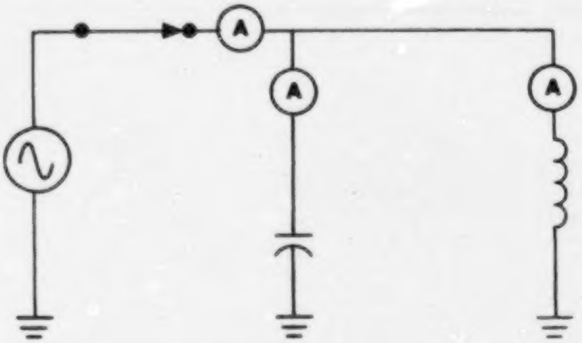
LESSON VI

Parallel Resonance Experiments

For this set of experiments you need NEAT Board 8, a signal generator, and your multimenter.

Observe that the NEAT Board has three parallel branches. We will use only the capacitive and inductive branches for our experiments. Also observe that in the inductive branch, by changing S-803, we have an option of adding one of three different valued resistors or of not having any resistance in series with the inductive leg.

The basic circuit for the experiments looks like this:



Notice that ammeter connections are indicated for I_{LINE}, I_C, and I_L. Your meter leads will be connected in the common test points T_P 806 and T_P 807 on the NEAT board, and the setting on S-802 will determine which current measurement is being taken. For example, if S-802 is in the M-(meter) 801 position, you will be reading line current as indicated by the meter face M-801.

Now perform the experiments.

Step 1. Setting up NEAT Board

1. Set S-801 to the ON position.
2. Set S-806 to position A.
3. Set S-802 to M-801 position
4. Set S-803 to position A.
5. Set S-804 to the OFF position.

Step 2. Setting Up Audio Signal Generator

1. Put band selector in position C.
2. Select sine wave position.
3. Turn frequency control all the way counterclockwise.
4. Turn amplitude control all the way clockwise.
5. Plug in signal generator and turn on power switch. (The red power-available light should glow.) Allow a few minutes for warm-up.
6. Connect test leads to output jacks (black to bottom terminal).
7. Insert other end of test leads to NEAT board terminals T_p-802 and T_p-803 (black to T_p-803).

Step 3. Setting Up Simpson 260

1. Set up Simpson to read DC current on 10 ma scale.
2. Insert test leads from Simpson to terminals T_p-806 and T_p-807 (black to T_p-806).

Experiment 1 - Determining f_o by I_{LINE}

Step 1. Slowly raise the frequency control of the signal generator and observe the current reading on the meter.

Step 2. By manipulating the frequency control, tune the signal generator to the frequency where line current is minimum and keep it there. This is f_o.

What does the range dial indicate resonant frequency to be? ____________

If frequency is varied above or below f_o, does I_{LINE} increase or decrease? ______________

Notice f_o is approximately 4.2 KHz. I_{LINE} increases when applied frequency varies either above or below f_o.

Experiment 2 - Determining f_o by I_C and I_L

Step 1. Switch S-802 to M-802 position. You will now be reading I_C.

Record reading __________

Step 2. Switch S-802 to M-803 position. You will now be reading I_L.

Record reading __________

What condition exists when I_L equals I_C? __________

What other circuit quantities must be equal in order for I_L to equal I_C? __________

I_C = approximately 3.47 ma

I_L = approximately 3.47 ma

When I_L and I_C are equal, resonance exists.

In an ideal tank, if I_L equals I_C, then X_L equals X_C.

Experiment 3 - Changing Q

Step 1. Put S-802 back to the M-801 position. You are reading I_{LINE} again. Tune signal generator again to ensure you are still at f_o.

Step 2. While switching S-803 from position A to position B, C, and D to add different values of resistance, note the meter reading at each position.

Has I_{LINE} increased or decreased? __________

In what position is resistance greatest? __________

In what position is Q lowest? __________

In what position is I_{LINE} greatest? __________

I_{LINE} increased.

Resistance is greatest in position D.

Q is lowest in position D.

I_{LINE} is greatest in position D.

Step 3. Place S-803 back to position A.

Experiment 4 - Changing f_o by Varying Capacitance

Step 1. Place S-806 in position B.

Step 2. This places variable capacitor C-801 in the circuit. Turn C-801 fully counterclockwise.

Step 3. Adjust signal generator until I_{LINE} is minimum. What is new f_o? ____________

Step 4. Adjust C-801 to center or midrange position.

Step 5. Adjust signal generator until I_{LINE} is minimum. What is new f_o? ____________

Step 6. Adjust C-801 fully clockwise.

Step 7. Adjust signal generator for minimum line current.

Step 8. Record new f_o ____________.

13.5 KHz; 18 KHz; 25 KHz

NOTE: When adjusting for minimum I_{LINE}, down scale multimeter as you approach 0 I_{LINE}.

When you decreased capacitance did you cause X_C to increase or decrease? __________

When X_C changed, did I_C increase or decrease? __________

Why did you have to raise frequency to reach a new f_o? ______________

X_C increased

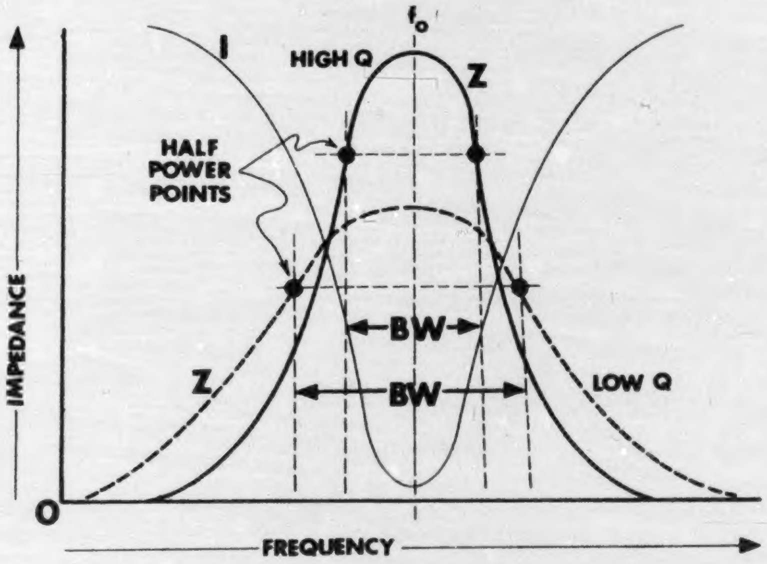
I_C decreased

The decrease in capacitance caused the circuit to resonate at a higher frequency.

TURN SIGNAL GENERATOR OFF. UNPLUG ALL TEST LEADS. UNPLUG SIGNAL GENERATOR.

Return the NEAT board and signal generator to the materials center.

Study the resonant characteristic curve for a parallel circuit shown below. How does it compare with your results?



AT THIS POINT, SEE YOUR LEARNING SUPERVISOR FOR FURTHER INSTRUCTIONS.

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2318	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4743	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6822	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
56	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
57	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8535	0.8545	0.8554	0.8563
58	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
59	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
60	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
61	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
62	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
63	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
64	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
65	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
66	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
67	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
68	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
69	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
70	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
	cos	0.3420	0.3404	0.3367	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
	cos	0.2924	0.2907	2.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
75	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
	cos	0.1564	1.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5953	7.6996	7.8062	7.9158	8.0285
83	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
deg	function	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°

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deg	func-tion	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°
84	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
85	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
86	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46
87	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
88	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
89	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0
deg	func-tion	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°